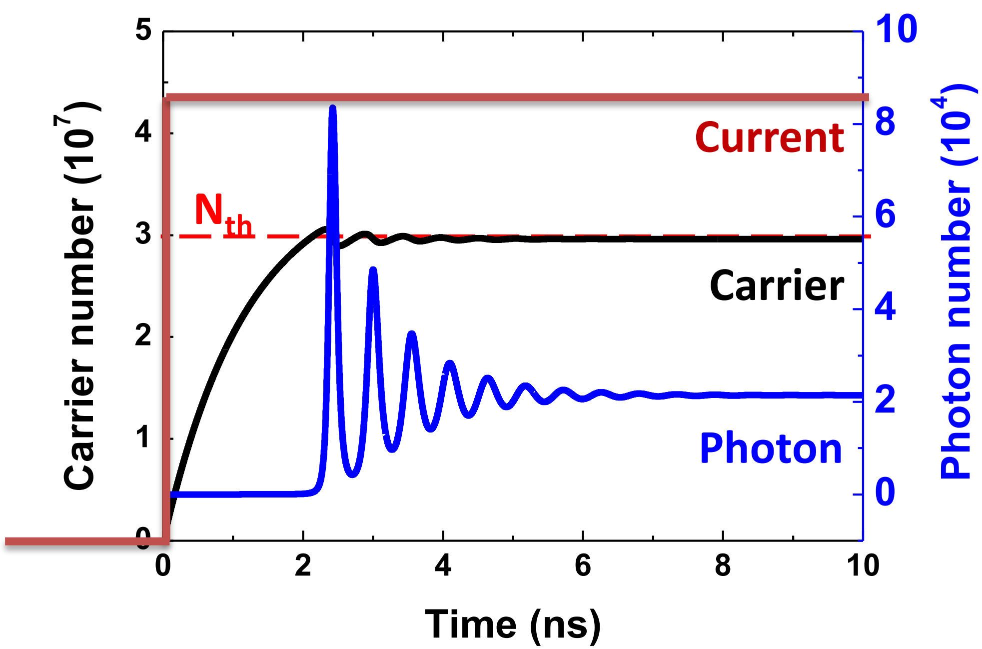
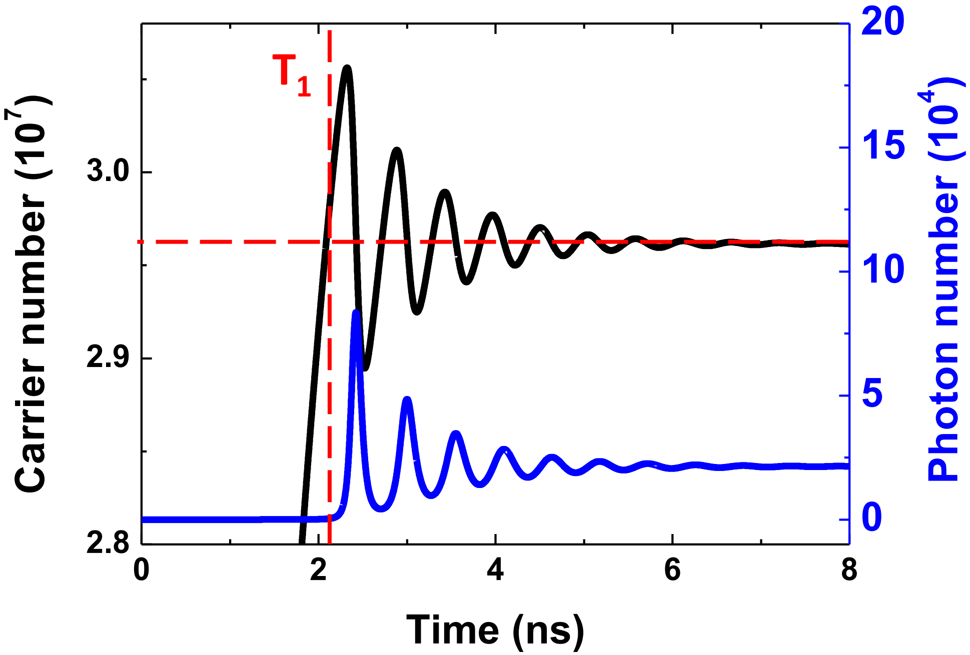
- Relaxation oscillation

Relaxation oscillation

- ❑ Relaxation oscillation: when the laser undergoes an external perturbation, the laser's coupled carrier and photon will mutually oscillates around their steady-state values. Eventually, the laser will reach the steady state in some time.
- ❑The laser is perturbed by a step-like current at time t=0 ns, the carrier and photon exhibits damped relaxation oscillations (阻尼振荡)



Relaxation oscillation

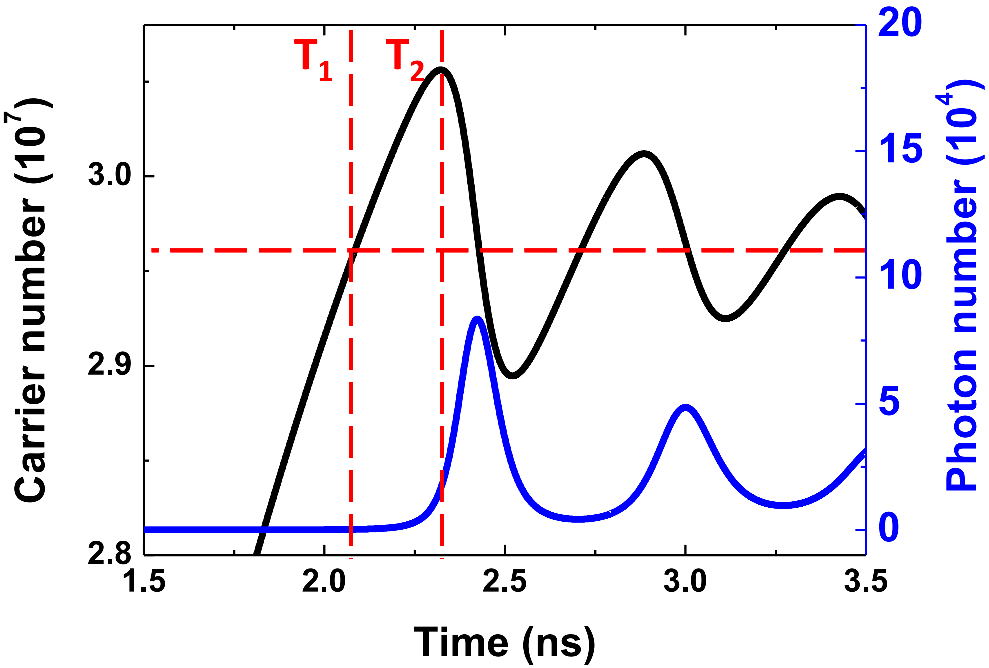


❑ Step1 ($0—T_1$)

✓ Carriers increase immediately with the step-like current, and reach the threshold at a delay time T_1, which is determined by the effective carrier lifetime.

✓ Photons begin to increase quickly starting from the delay time T_1.

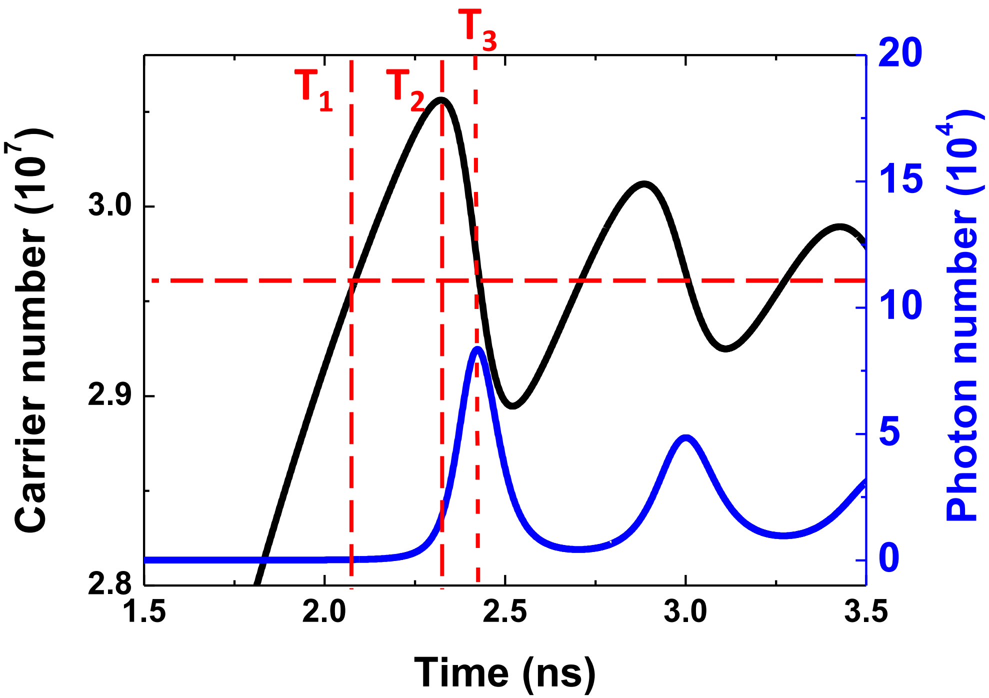
Relaxation oscillation



❑ Step2 (T_1—T_2)

✓ On one hand, the carriers remain increase due to the pump current. On the other hand, pupolation inversion $N>N_{th}$ leads to the increase of photon number. The increased photon number induces carrier saturation. Finally, the carrier number reaches maximum at a certain photon number at T_2.

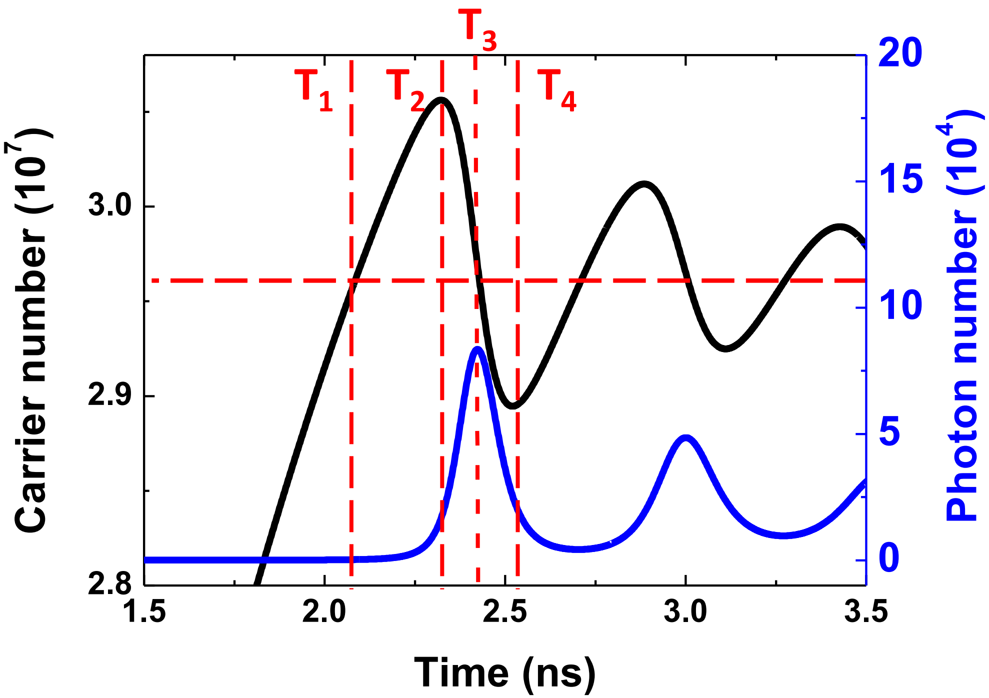
Relaxation oscillation



- Step3 (T_2—T_3)

✓The pupolation inversion $N>N_{th}$, so the photon number remains increase, leading to stronger carrier saturation effect. Therefore, the carrier number decreases. When the carrier number reaches the threshold, the photon number reaches the maximum at T_3.

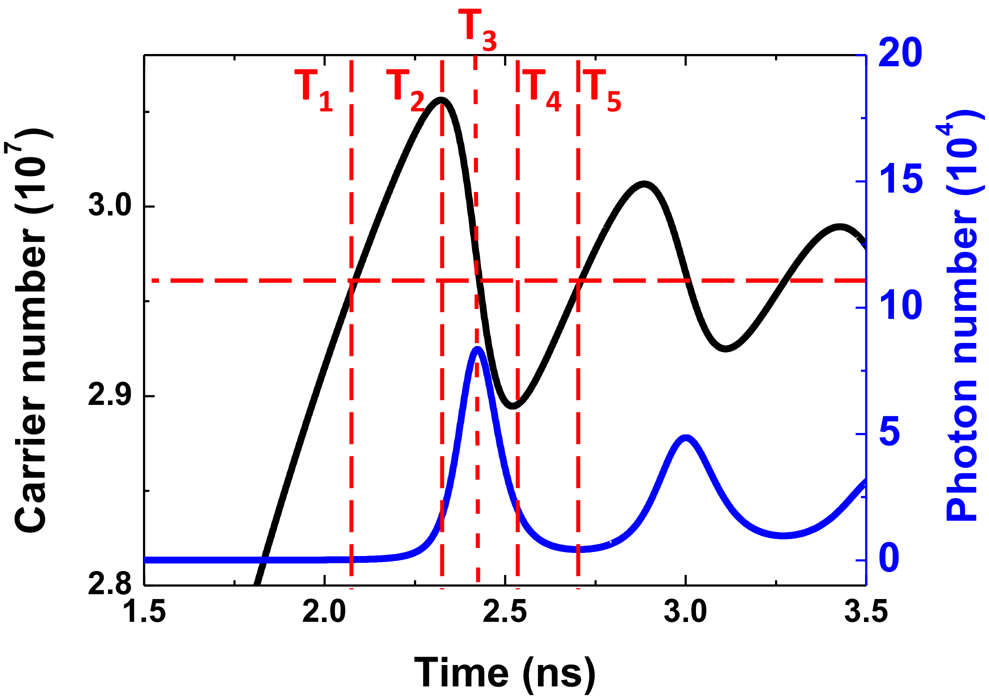
Relaxation oscillation



❑ Step4 (T_3—T_4)

✓ The photons remain reduces the carrier number due to the saturation effect, and making $N<N_{th.}$ On the other hand, because it is below threshold, the photon undergoes loss leading to the photon number decrease. The saturation effect becomes weak, and finally is balanced by the pump process at T_4.

Relaxation oscillation



❑ Step5 (T_4—T_5)

✓ The photon number remains decreasing since below threshold. The pump process accumulates carriers, and again reaches the threshold at T_5.

❑ The question is what is the relaxation period? How long it is needed to reach the steady state?

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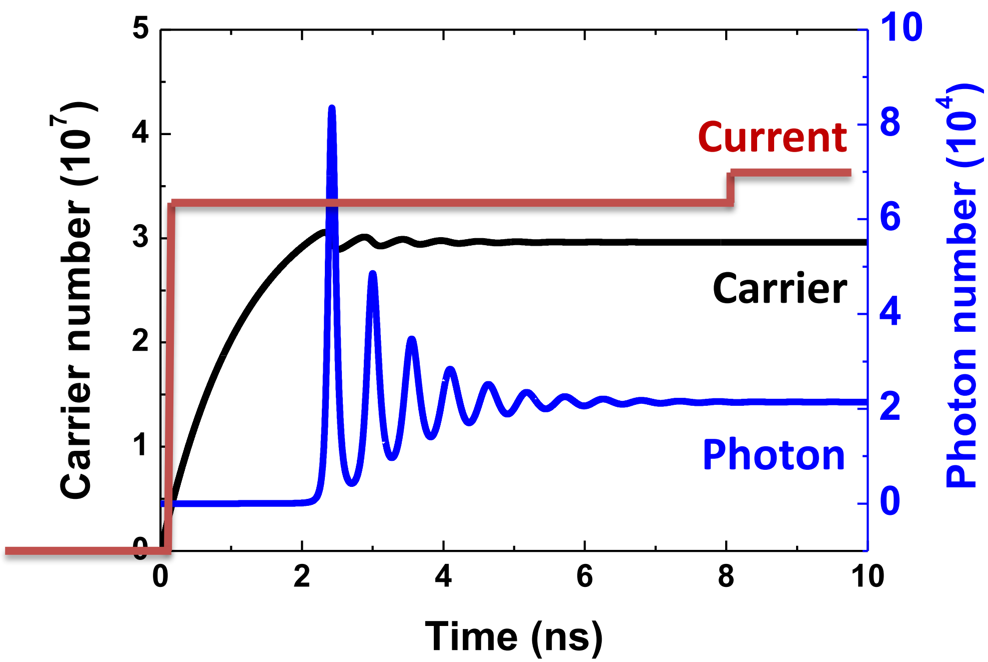
Small-signal (linearized) analysis

❑ When the pertubation induced carrier and photon variation is much smaller than their steady-state values, it is the small-signal perturbation.



Small-signal (linearized) analysis

The rate equations

$$\frac{dN}{dt} = R_p - \mathrm{v_g} g N_P - \frac{N}{\tau_{sp}}$$

$$\frac{dN_P}{dt} = \mathrm{v_g} g N_P - \frac{N_P}{\tau_p} + \beta \frac{N}{\tau_{sp}}$$

Steady-state solution

$$R_p - \mathrm{v_g} g_0 N_{P0} - \frac{N_0}{\tau_{sp}} = 0$$

$$\mathrm{v_g} g_0 N_{P0} - \frac{N_{P0}}{\tau_p} + \beta \frac{N_0}{\tau_{sp}} = 0$$

Small-signal perturbation

$$R_p(t) = R_{p0} + \delta R_p(t)$$

$$N(t) = N_0 + \delta N(t)$$

$$N_p(t) = N_{p0} + \delta N_p(t)$$

$$\delta N(t) << N_0; \ \delta N_p(t) << N_{p0}$$

The rate equations

$$\frac{d\delta N}{dt} = R_p + \delta R_p - \mathrm{v_g} \sigma (N_0 + \delta N)(N_{p0} + \delta N_p) - \frac{(N_0 + \delta N)}{\tau_{sp}}$$

$$\frac{d\delta N_P}{dt} = \mathrm{v_g} \sigma (N_0 + \delta N)(N_{p0} + \delta N_p) - \frac{(N_{p0} + \delta N_p)}{\tau_p} + \beta \frac{(N_0 + \delta N)}{\tau_{sp}}$$

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Small-signal (linearized) analysis

The rate equations

$$\frac{d\delta N}{dt}=\left(R_p-\mathrm{v_g}\sigma N_0 N_{p0}-\frac{N_0}{\tau_{sp}}\right)+\left[\delta R_p\text{-}\mathrm{v_g}\sigma\left(N_{p0}\delta N+N_0\delta N_p\right)-\frac{\delta N}{\tau_{sp}}\right]\text{-}\left(\mathrm{v_g}\sigma\delta N\delta N_p\right)$$

$$\frac{d\delta N_P}{dt}=\left(\mathrm{v_g}\sigma N_0 N_{p0}-\frac{N_{p0}}{\tau_p}+\beta\frac{N_0}{\tau_{sp}}\right)+\left[\mathrm{v_g}\sigma\left(N_{p0}\delta N+N_0\delta N_p\right)-\frac{\delta N_p}{\tau_p}+\beta\frac{\delta N}{\tau_{sp}}\right]+\left(\mathrm{v_g}\sigma\delta N\delta N_p\right)$$

- Ignore high-order terms, we get the linearized equations

Linearized rate equations

$$\frac{d\delta N}{dt}=\delta R_p-\mathrm{v_g}\sigma\left(N_{p0}\delta N+N_0\delta N_p\right)-\frac{\delta N}{\tau_{sp}}$$

$$\frac{d\delta N_P}{dt}=\mathrm{v_g}\sigma\left(N_{p0}\delta N+N_0\delta N_p\right)-\frac{\delta N_p}{\tau_p}+\beta\frac{\delta N}{\tau_{sp}}$$

- Above threshold,

Linearized rate equations

$$\mathrm{v_g}\sigma N_0\approx\frac{1}{\tau_p}$$

Linearized rate equations

$$\frac{d\delta N}{dt}=\delta R_p-\mathrm{v_g}\sigma N_{p0}\delta N-\frac{\delta N_p}{\tau_p}-\frac{\delta N}{\tau_{sp}}$$

$$\frac{d\delta N_P}{dt}=\mathrm{v_g}\sigma N_{p0}\delta N+\beta\frac{\delta N}{\tau_{sp}}$$

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Small-signal (linearized) analysis

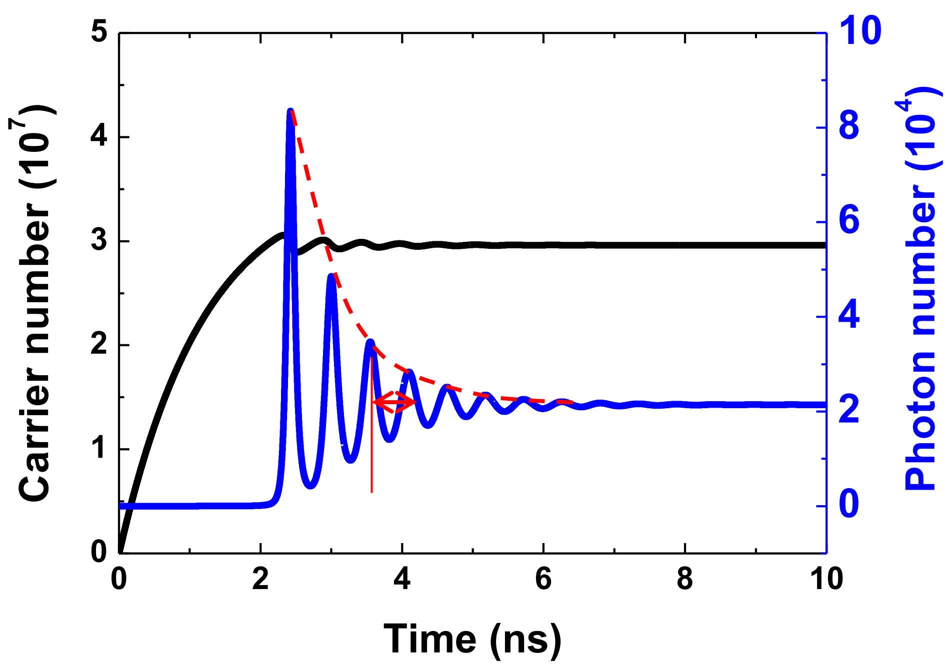
- The solution of the linearized rate equation is of the form.

$$\delta N(t) = N_X \exp\left(-\frac{\Gamma}{2}t\right)\sin(\omega_R t)$$

$$\delta N_p(t) = N_{pX} \exp\left(-\frac{\Gamma}{2}t\right)\cos(\omega_R t)$$

ω_R is the relaxation resonance frequency, determining the oscillation period.

Γ is the damping factor, determining the oscillation decay speed .



Linearized rate equations

$$\frac{d\delta N}{dt} = \delta R_p - \mathrm{v_g}\sigma N_{p0}\delta N - \frac{\delta N_p}{\tau_p} - \frac{\delta N}{\tau_{sp}}$$

$$\frac{d\delta N_P}{dt} = \mathrm{v_g}\sigma N_{p0}\delta N + \beta\frac{\delta N}{\tau_{sp}}$$

Small-signal (linearized) analysis

- The carrier and photon variations are in the form

$$\delta R_p(t) = \delta r_p \exp(j\omega t)$$
$$\delta N(t) = \delta n \exp(j\omega t)$$
$$\delta N_p(t) = \delta n_p \exp(j\omega t)$$

Linearized rate equations

$$j\omega\delta n \exp(j\omega t) = \delta r_p \exp(j\omega t) - \mathrm{v}_g \sigma N_{p0} \delta n \exp(j\omega t) - \frac{\delta n_p}{\tau_p} \exp(j\omega t) - \frac{\delta n \exp(j\omega t)}{\tau_{sp}}$$

$$j\omega\delta n_P \exp(j\omega t) = \mathrm{v}_g \sigma N_{p0} \delta n \exp(j\omega t) + \beta \frac{\delta n \exp(j\omega t)}{\tau_{sp}}$$

Linearized rate equations

$$j\omega\delta n = \delta r_p - \mathrm{v}_g \sigma N_{p0} \delta n - \frac{\delta n_p}{\tau_p} - \frac{\delta n}{\tau_{sp}}$$

$$j\omega\delta n_P = \mathrm{v}_g \sigma N_{p0} \delta n + \beta \frac{\delta n}{\tau_{sp}}$$

Linearized rate equations

$$\frac{d\delta N}{dt} = \delta R_p - \mathrm{v}_g \sigma N_{p0} \delta N - \frac{\delta N_p}{\tau_p} - \frac{\delta N}{\tau_{sp}}$$

$$\frac{d\delta N_P}{dt} = \mathrm{v}_g \sigma N_{p0} \delta N + \beta \frac{\delta N}{\tau_{sp}}$$

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Small-signal (linearized) analysis

- The linearized equation in the matrix form

$$\begin{bmatrix} j\omega + \mathrm{v_g}\sigma N_{p0} + \dfrac{1}{\tau_{sp}} & \dfrac{1}{\tau_p} \\ -\mathrm{v_g}\sigma N_{p0} - \dfrac{\beta}{\tau_{sp}} & j\omega \end{bmatrix} \begin{bmatrix} \delta n \\ \delta n_P \end{bmatrix} = \begin{bmatrix} \delta r_p \\ 0 \end{bmatrix}$$

- The determinant of the matrix

$$\Delta = \begin{vmatrix} j\omega + \gamma_{nn} & \gamma_{np} \\ -\gamma_{pn} & j\omega \end{vmatrix}$$
$$= \gamma_{np}\gamma_{pn} - \omega^2 + j\omega\gamma_{nn}$$
$$= \omega_R^2 - \omega^2 + j\omega\Gamma$$

- The resonance frequency

$$\omega_R^2 = \gamma_{np}\gamma_{pn}$$
$$= \frac{1}{\tau_p}\left(\mathrm{v_g}\sigma N_{p0} + \frac{\beta}{\tau_{sp}}\right)$$

- The damping factor

$$\Gamma = \gamma_{nn}$$
$$= \mathrm{v_g}\sigma N_{p0} + \frac{1}{\tau_{sp}}$$

Small-signal (linearized) analysis

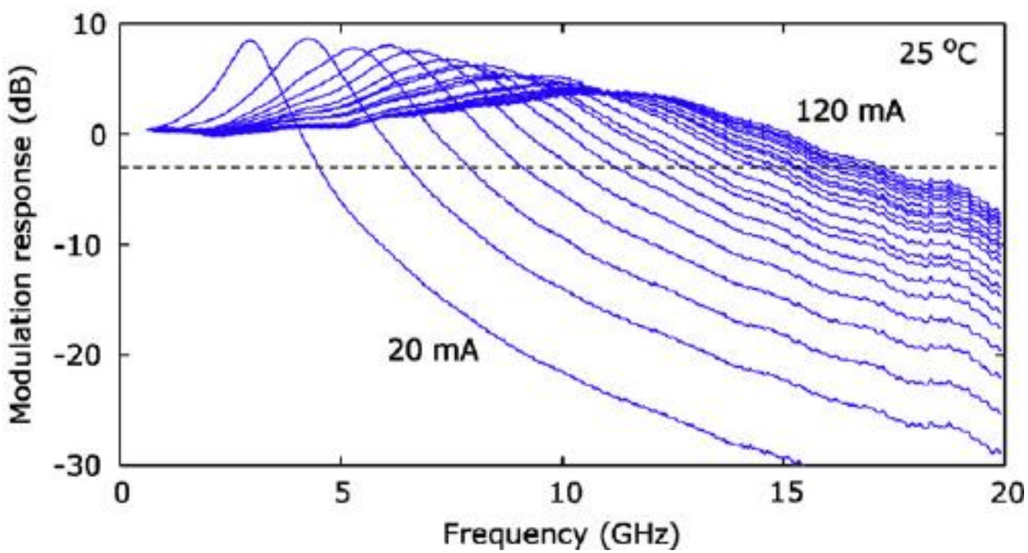
- The solutions of the linearized equations

$$\delta n = \frac{1}{\Delta}\begin{bmatrix} \delta r_p & \dfrac{1}{\tau_p} \\ 0 & j\omega \end{bmatrix}$$

$$\delta n_P = \frac{1}{\Delta}\begin{bmatrix} j\omega + \mathrm{v_g}\sigma N_{p0} + \dfrac{1}{\tau_{sp}} & \delta r_p \\ -\mathrm{v_g}\sigma N_{p0} - \dfrac{\beta}{\tau_{sp}} & 0 \end{bmatrix}$$

$$\frac{\delta n}{\delta r_p} = \frac{1}{\Delta} j\omega$$

$$\frac{\delta n_P}{\delta r_p} = \frac{1}{\Delta}\left(\mathrm{v_g}\sigma N_{p0} + \frac{\beta}{\tau_{sp}}\right)$$



Examples 8.1, 8.2

$$\begin{bmatrix} j\omega + \mathrm{v_g}\sigma N_{p0} + \dfrac{1}{\tau_{sp}} & \dfrac{1}{\tau_p} \\ -\mathrm{v_g}\sigma N_{p0} - \dfrac{\beta}{\tau_{sp}} & j\omega \end{bmatrix}\begin{bmatrix} \delta n \\ \delta n_P \end{bmatrix} = \begin{bmatrix} \delta r_p \\ 0 \end{bmatrix}$$

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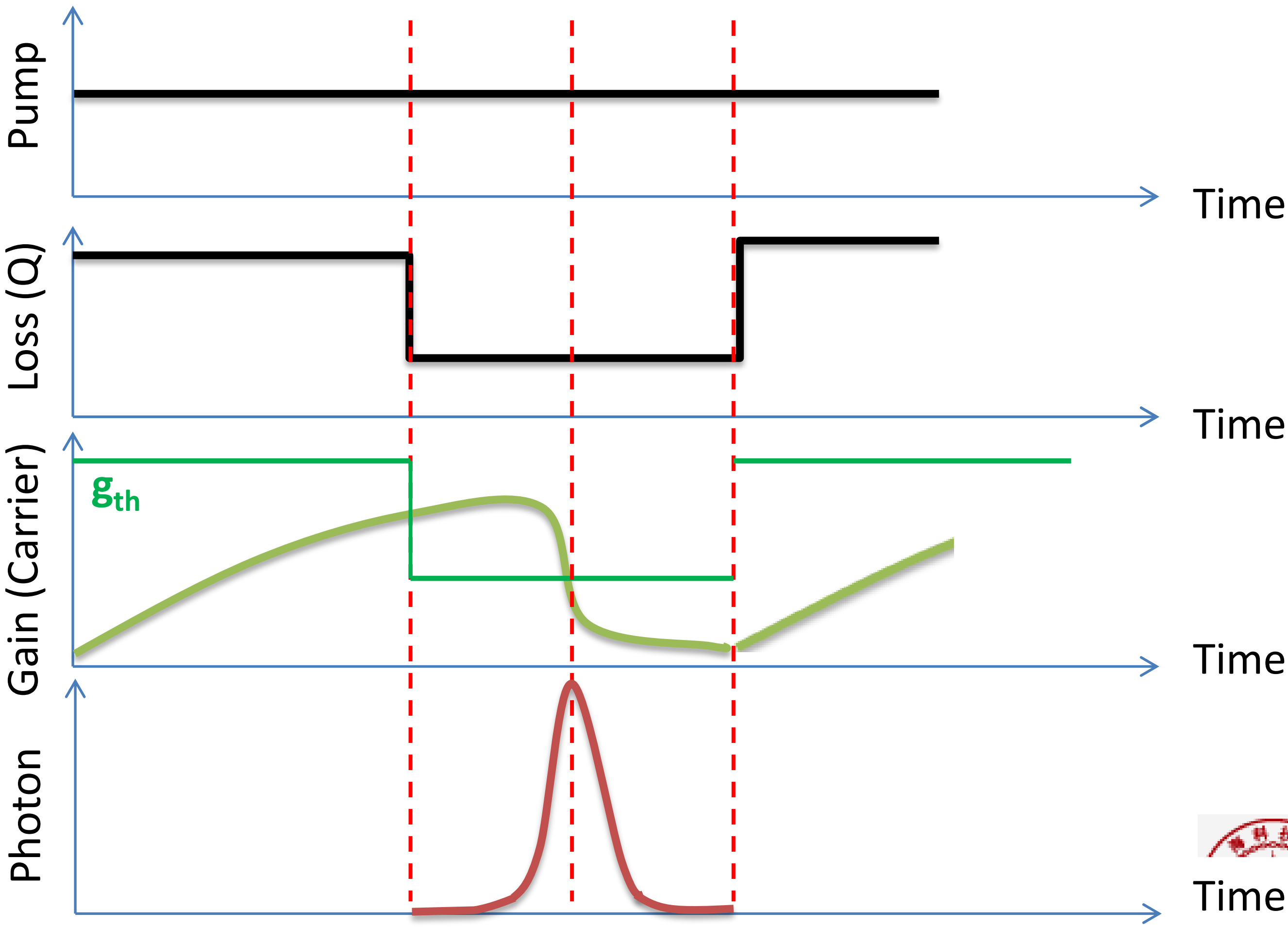
Chapter 8_L21

Q switching technique

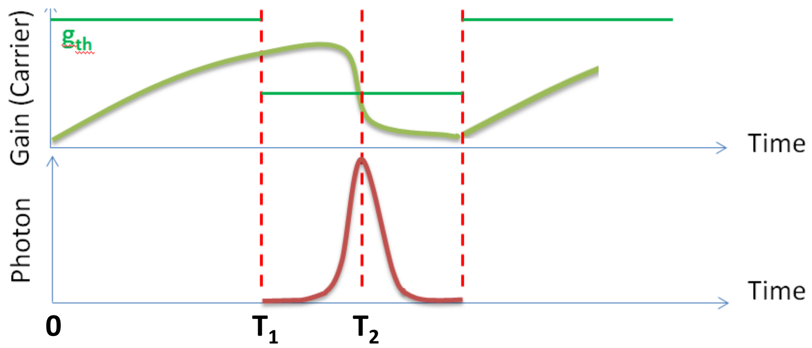
- ✓ Electro-optic Q switch
- ✓ Acousto-optic Q switch
- ✓ Saturable-absorber Q switch

Q switching

❑ Q switching is a technique for obtaining energetic short (but not ultrashort) pulses from a laser by modulating the intracavity losses and thus the Q factor of the laser resonator. The technique is mainly applied for the generation of nanosecond pulses of high energy and peak power with solid-state bulk lasers.



Q switching



❑ Step1 (0—T_1)

✓ The cavity loss is at a high level, the lasing canot occur. So the pump accumulates carriers in the gain medium. The maximum stored carrier energy (amount) is limited by the carrier lifetime (spontaneous emission T_{sp}). Therefore, this energy storing period is limited as $T_1 < T_{sp}$, usually on the order of ms (0.1 ms– 1.0 ms).

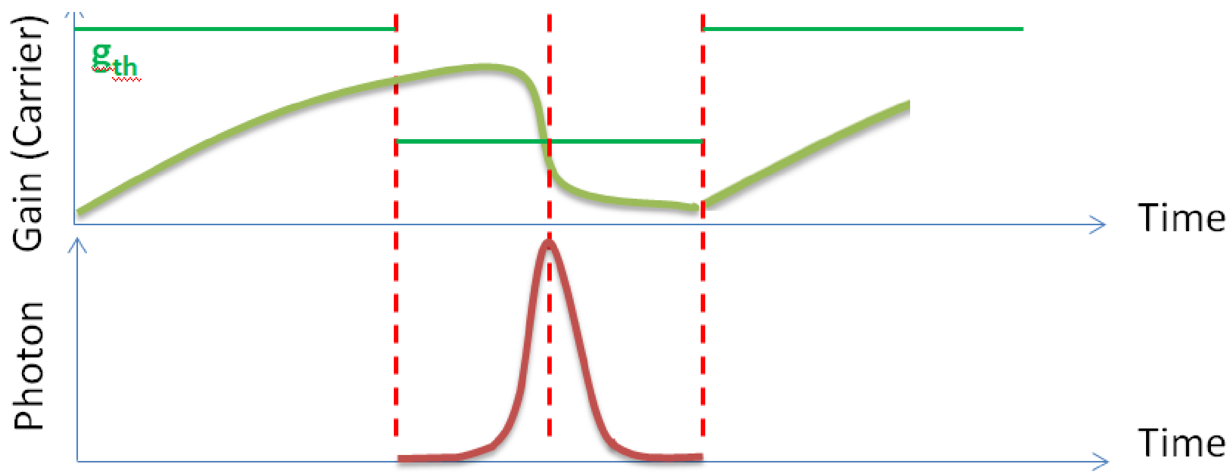
❑ Step2 (T_1—T_2)

✓ The cavity loss is suddenly reduced to a low level at T_1, then the carrier population is much larger than the threshold value. So the power of the laser radiation build up very quickly in the cavity.

✓ When the power becomes strong, the gain reduces due to the saturation effect. The peak of the laser pulse is reached when the carrier population reduces to the threshold. Below threshold, the pulse power decays, while it remains depletes the carrier populations.

Q switching

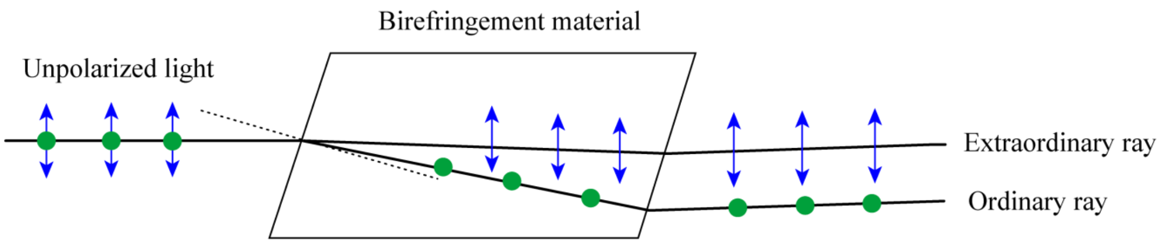
- The laser pulse duration is usually on the order of ns (1.0 ns–50 ns). The pulse energy ranges from mJ up to kJ, thereby the pulse peak power (10^{4}– 10^{12} W) is much higher than the CW laser power with the same pump. The pulse repetition rate is usually in the range of 1.0 kHz– 100 kHz.



- Q switching methods include active Q switching using electro-optic Q-switches or acousto-optic Q-switches, and passive Q switching using saturable absorber (optical nonlinearity).

Electro-optic Q switching

❑ Electro-optic effect (Pockels effect): For some nonlinear crystals such as KD^*P, $LiNbO_3$, when a voltage is applied, it will exhibit birefrigence effect. The birefrigence effect splits the electric field E into an ordinary wave (o-wave <p or s>, Ex) and an extrodinary wave (e-wave <s or p>, Ey), which are perpendicular to each other. The o-wave and the e-wave have different refractive indices n_o and n_e. The two refractive indices change with the applied voltage.



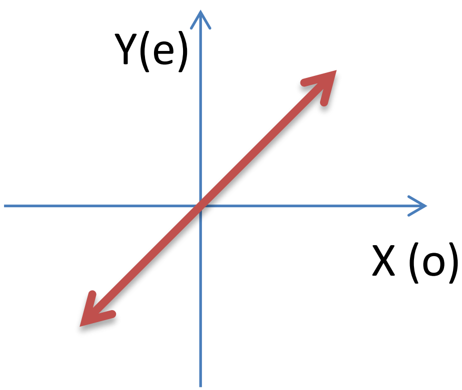
❑ When the field propagates along the optical axis of the crystal, o-wave and e-wave will be on the same direction.

❑ The device with Pockels effect is called Pockels cell. When an linearly polarized electric field passes through the cell with 45^0 to the birefrigence axises, the two wave components will have a phase difference:

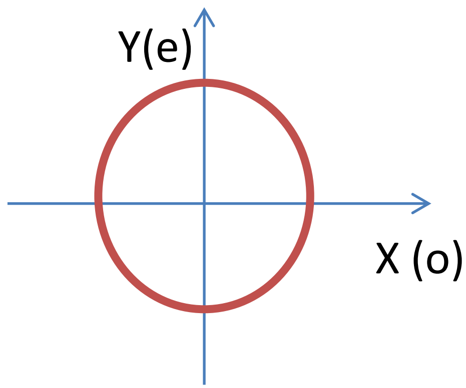
$$\boxed{\Delta\phi_{eo} = k\left(n_e - n_o\right)L}$$

Electro-optic Q switching

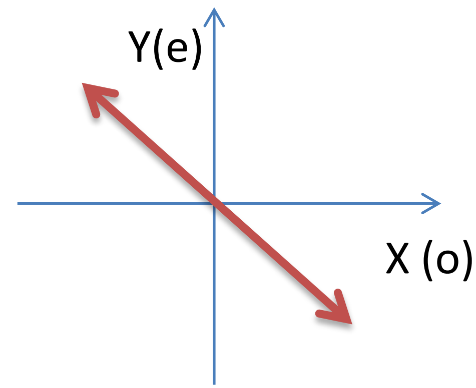
❑ If the phase difference is 90^0, the polarization of the electric field will become circular. If the phase difference is 180^0, the electric field is still linearly polarized, but the direction is perpendicular to the original direction.



Linear E-field



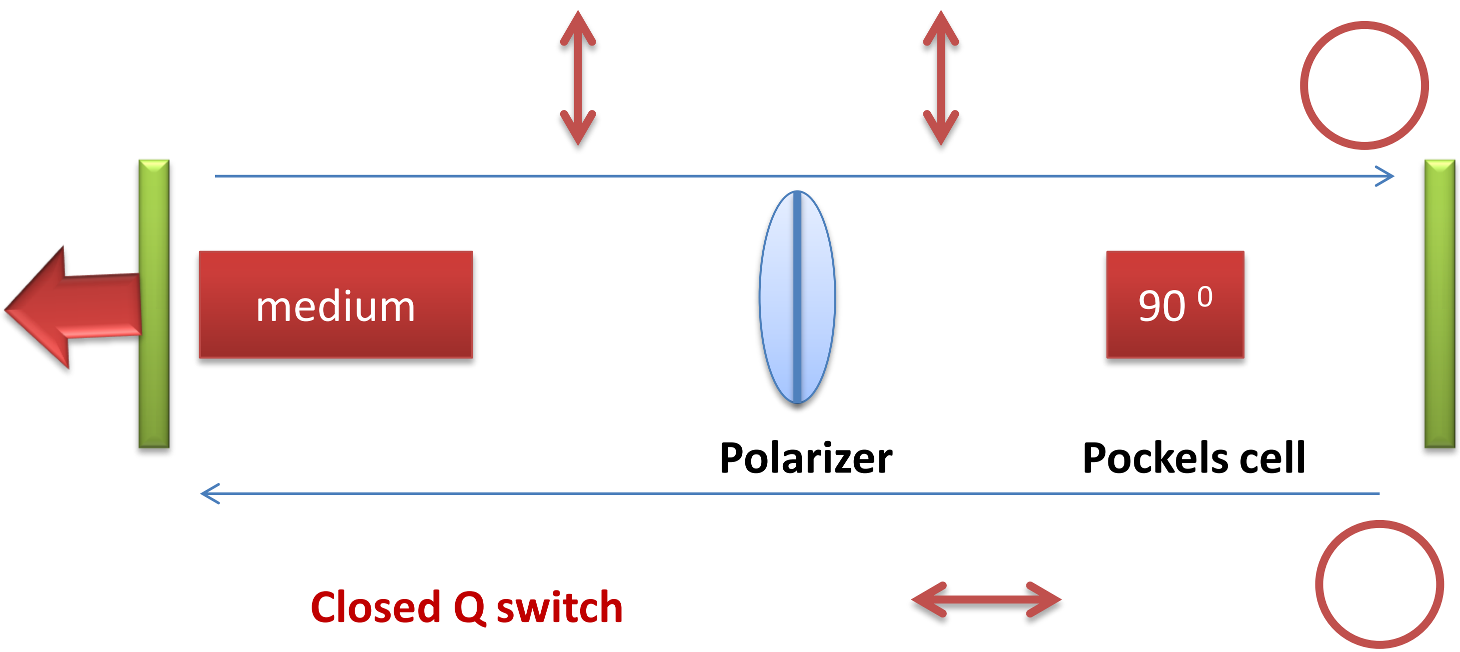
90^0 phase shift



180^0 phase shift

Electro-optic Q switching

❑ In Q switching, the Pockels cell is operated to exhibit a 90^0 phase shift as the following experimental setup.

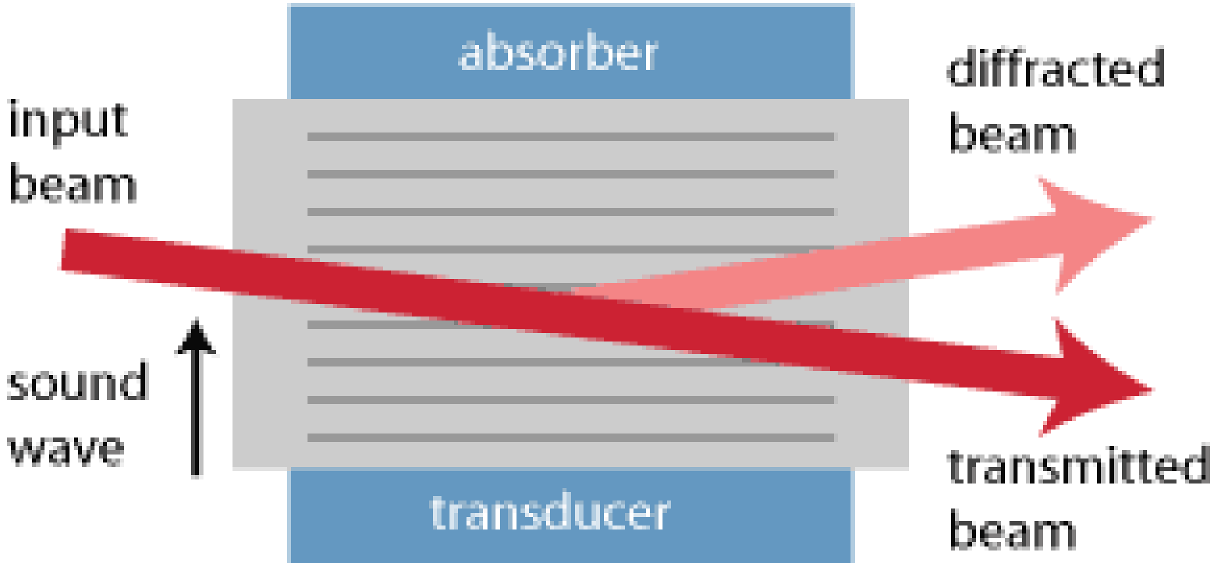


❑ The required volatage to produce 90^0 phase shift is called «quarter-wave volage», which may range between 1 kV and 5 kV.

❑ The switch of the voltage must be fast (typically <20 ns), smaller than the laser pulse build-up time.

Acousto-optic Q switching

- When a traveling acoustic wave is applied to some crystal like quartz, the strain induced by the acoustic wave results in local changes of the material refractive index through the photoelastic effect. This periodic change of refractive index acts as a phase grating with period equal to the acoustic wavelength, amplitude proportional to the sound amplitude, and which is traveling at the sound velocity in the medium (traveling-wave phase grating). Its effect is to diffract a fraction of the incident beam out of the direction.



Acousto-optic Q switching

❑ The requirements of the operation (Bragg diffraction)

$$L >> \frac{n_r \lambda_a^2}{2\pi\lambda}; \quad \theta_{in} = \frac{\lambda}{2\lambda_a}$$

$$\theta_{out} = 2\theta_{in};$$

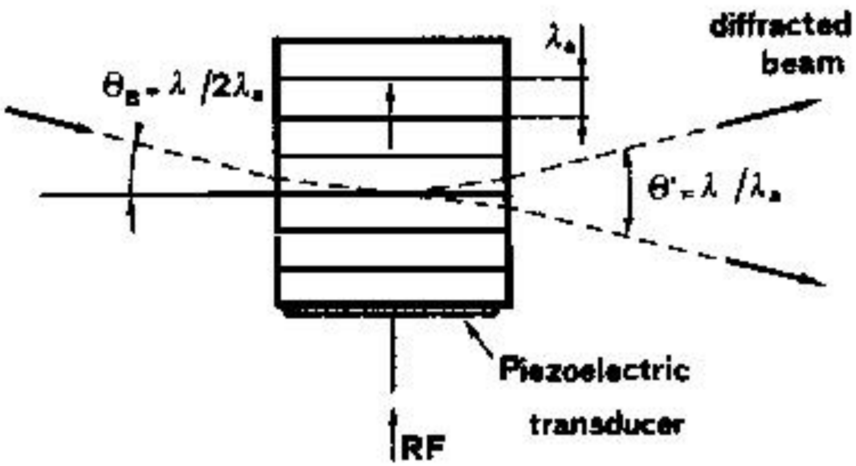
$$I_{out} = I_{in} \sin^2\left(\frac{\Delta\varphi_L}{2}\right)$$

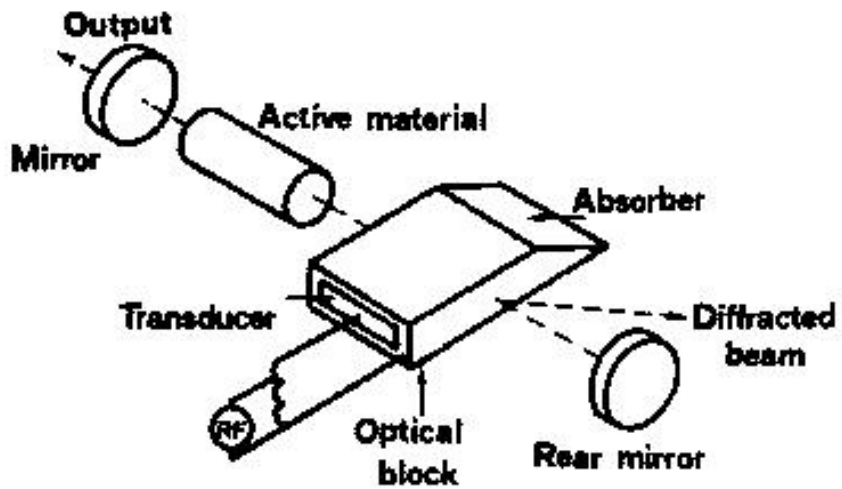
✓ The voltage applited to the AO switch is around 100 V.

✓The AO switch induced loss is smaller than that by the EO switch, so can not be used to very high energy laser pulse generation.

Examples 8.3

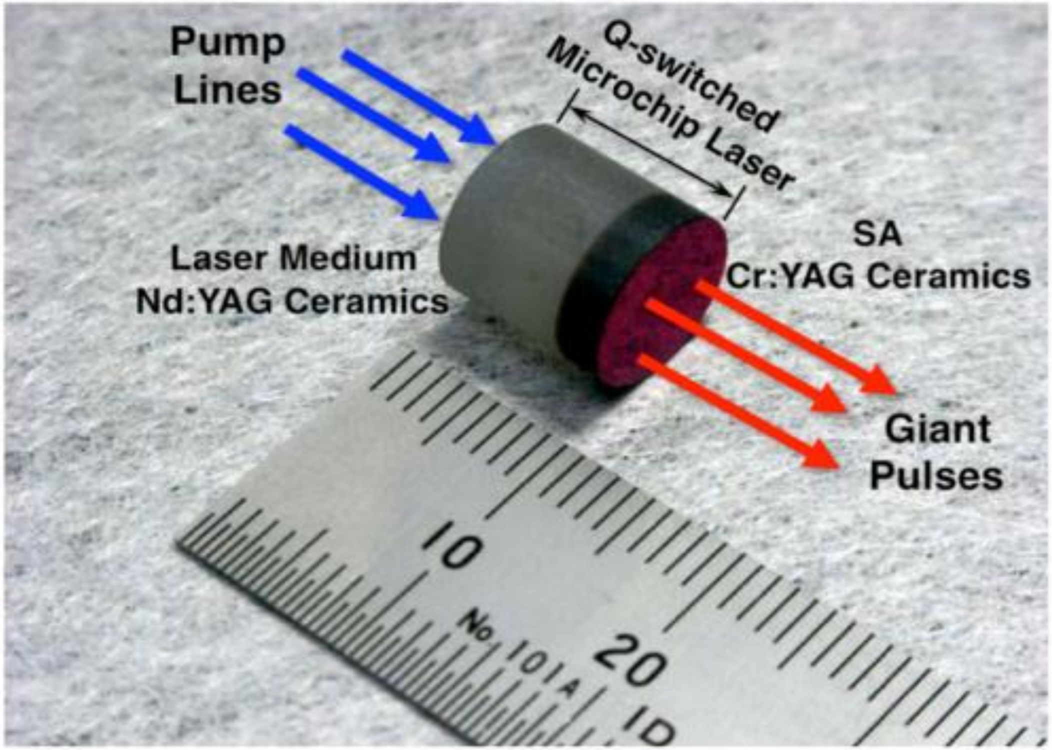
$\Delta\varphi_L$ is the phase shift of the light due to the phase grating





Saturable-absorber Q switch

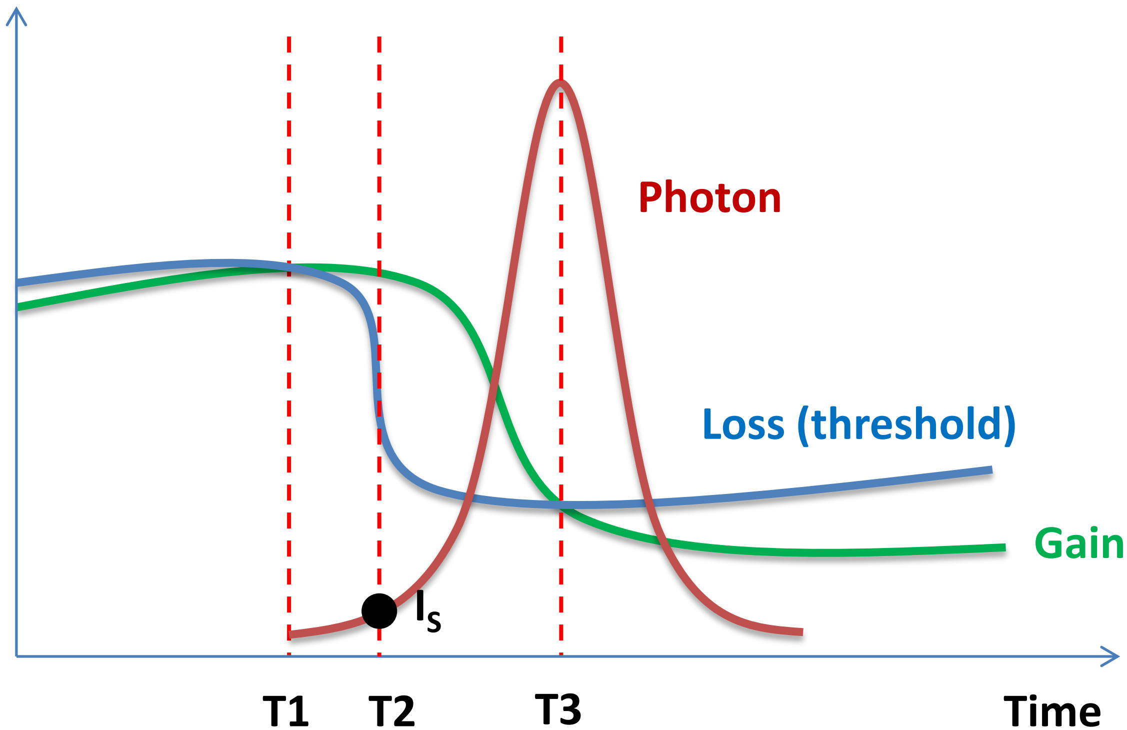
- The absorber can be treated as a two-level system with most carriers in the ground state, and with a large absorption cross section (small saturation intensity, I_s), such as Cr^{4+}:YAG. The absorber exhibits a high loss (absorption) when the incident light is weak, while becomes almost transparent (no absorption) when the light is strong ($>I_s$).



$$\alpha = \frac{\alpha_0}{1 + I / I_s}$$

$$I_s = \frac{h\nu}{2\sigma_a \tau_{sp}}$$

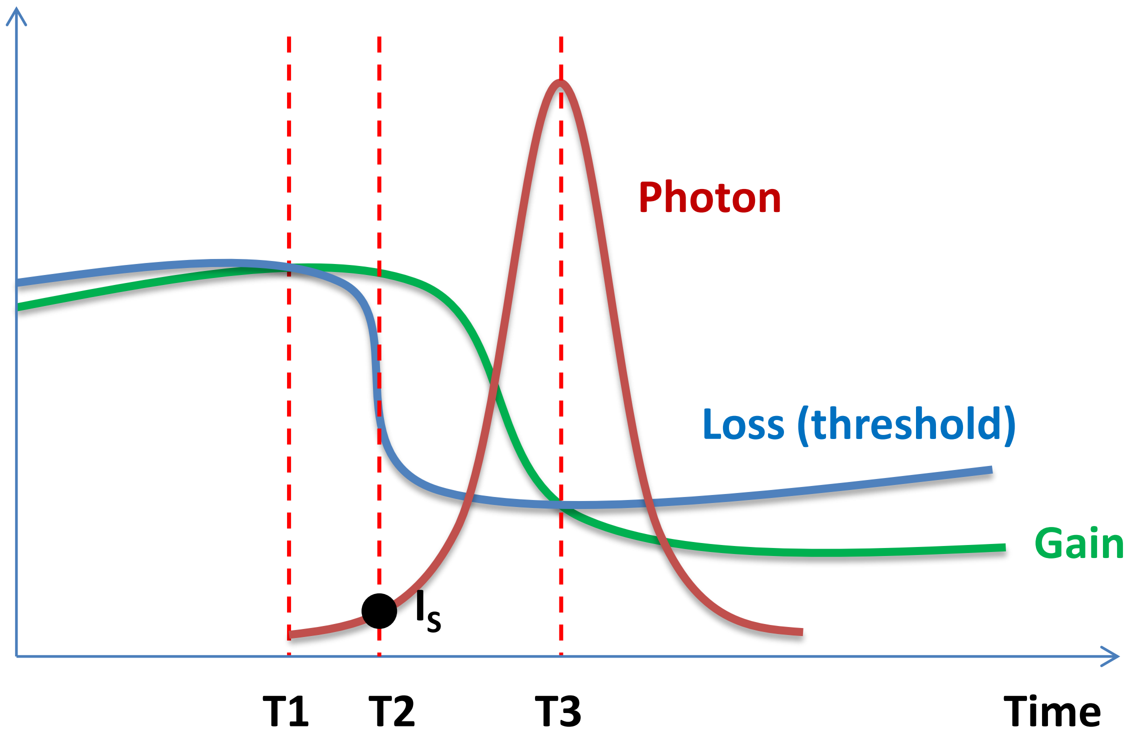
Saturable-absorber Q switch



- Step1 ($0—T_1$)

The aborber has a high loss due to the absorption. The laser is below threshold. At T1, the pump increases the gain to the threshold, and the laser starts to lasing.

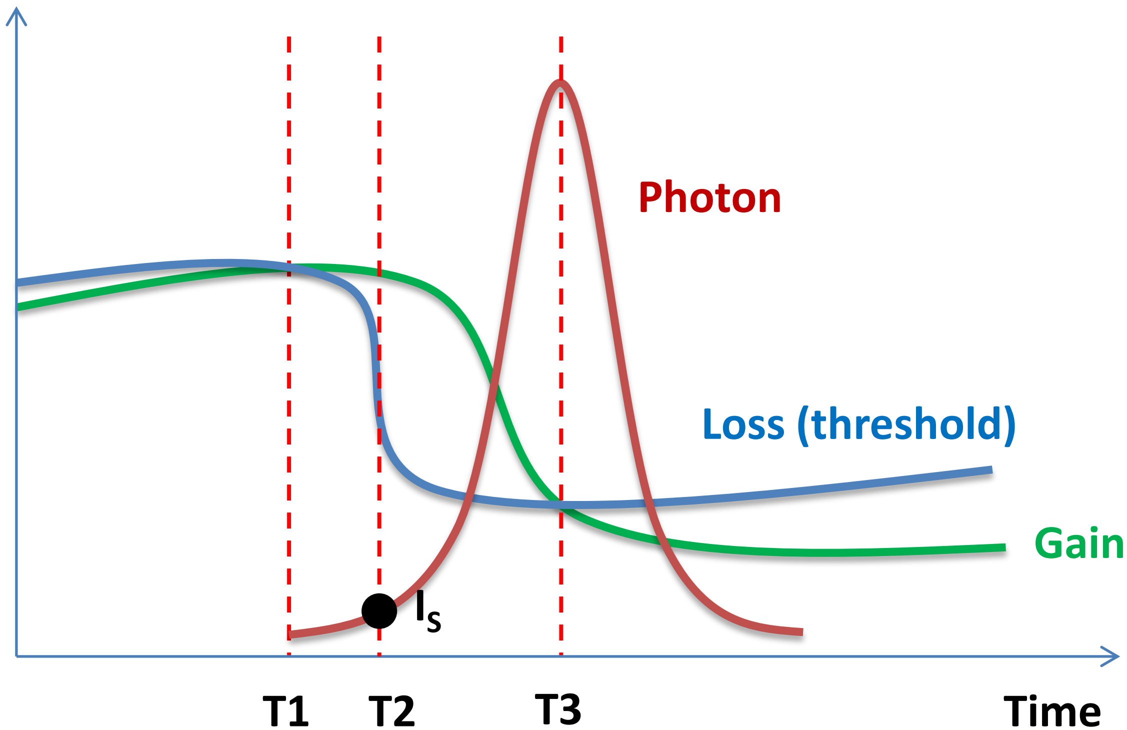
Saturable-absorber Q switch



- Step2 (T_1—T_2)

The photon reduces the absorption coefficient due to the absorption saturation effect. At T2, the photon intensity reaches the absorber saturation intensity, and the absorber becomes almost transparent with very low loss. Thus, the gain is far beyond the threshold.

Saturable-absorber Q switch



- Step3 (T_2—T_3)

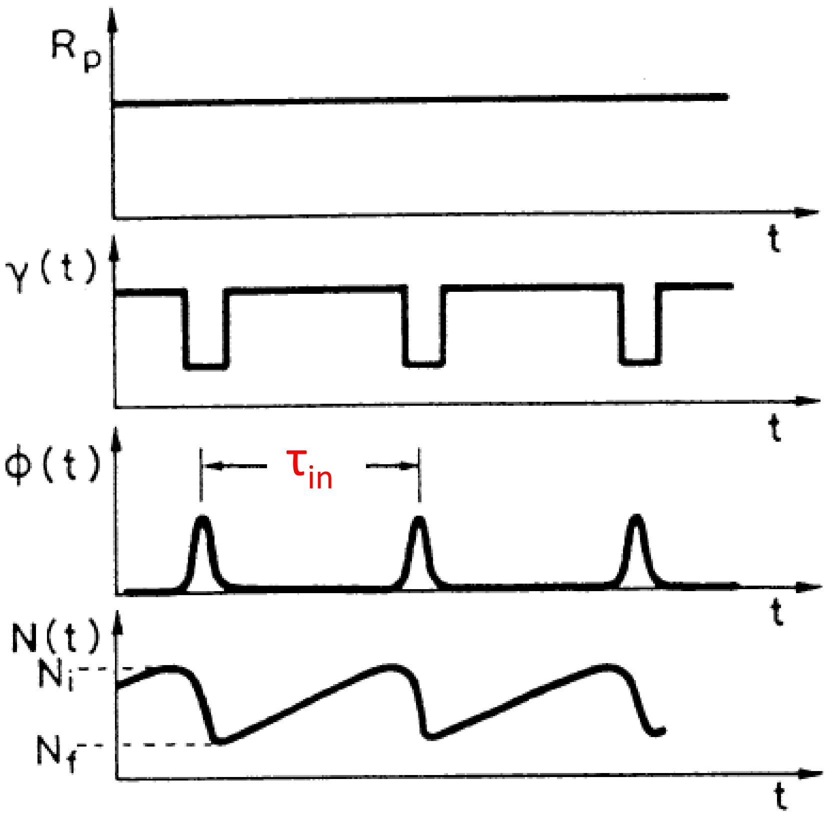
Since the gain is far beyond the threshold, the photon intensity increases rapidly. The gain reduces due to the gain saturation effect. The photon reaches the peak when the gain reduces to be the same as the loss.

Saturable-absorber Q switch

- Compared with active Q switching, passive Q switching is simple and cost-effective (eliminating the modulator and its electronics), and is suitable for very high pulse repetition rates. However, the pulse energies are typically lower. It may also be a disadvantage that the pulse energy and duration are often more or less independent of the pump power, which only determines the pulse repetition rate.

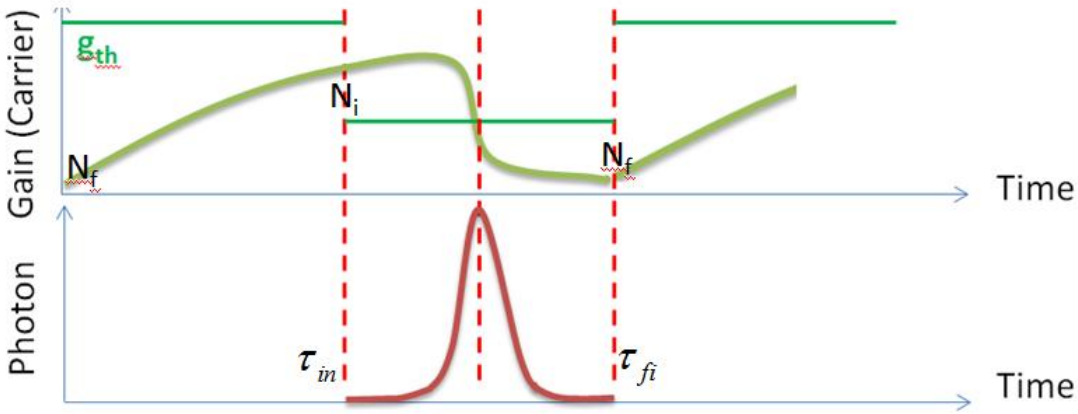
- Q-switching laser pulse characteristics

Q switching characteristics



- The maximum time τ_{in} for restoring carrier population (high loss) must be smaller than the carrier lifetime of the upper lasing state τ_{sp} , or else carriers will be killed by the spontaneous emission.
- Therefore, the laser pulse repetition rate ($1/\tau_{in}$) is usually from 1 kHz to up to 100 kHz.
- The questions for the laser pulse:
  - ✓ What is the pulse peak power?
  - ✓ What is the pulse energy?
  - ✓ What is the pulse duration?
  - ✓ What is the pulse buid up time?

Pulse peak power



- When the Q switch changes from closed to open at τ_{in}, the initial conditions:
  - ✓ The photon density: N_{Pi}~0
  - ✓ The carrier density: N_i

- During $[0, \tau_{in}]$, the rate equations

$$N_p(t) = 0$$

$$\frac{dN}{dt} = R_p - \frac{N}{\tau_{sp}} \Rightarrow$$

$$N(t) = \left(N_f - R_p\tau_{sp}\right)\exp\left(-\frac{t}{\tau_{sp}}\right) + R_p\tau_{sp}$$

$$N_i = \left(N_f - R_p\tau_{sp}\right)\exp\left(-\frac{\tau_{in}}{\tau_{sp}}\right) + R_p\tau_{sp}$$

$$N_{pi} = 0$$

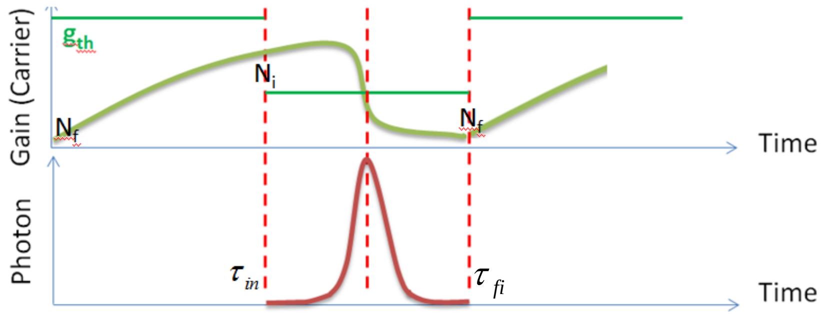
- During $[\tau_{in}, \tau_{fi}]$, the laser pulse duration is on the order of ns, it is so fast that the pump and the spontaneous emission have little impact on the carrier population during this period, and thus can be ignored.

$$\frac{dN}{dt} = -\mathrm{v}_g g N_P$$

$$\frac{dN_P}{dt} = \mathrm{v}_g g N_P - \frac{N_P}{\tau_p}$$

Pulse peak power

- At the peak of the laser pulse,



- The carrier density

$$\frac{dN_P}{dt} = \mathrm{v_g} g N_P - \frac{N_P}{\tau_p} = 0 \Rightarrow$$

$$N^{peak} = \frac{1}{\sigma \mathrm{v_g} \tau_p} = N_{th}$$

- The photon density

$$\frac{dN_P}{dN} = \frac{1}{\tau_p \mathrm{v_g} g} - 1 \Rightarrow$$

$$\frac{dN_P}{dN} = \frac{N_{th}}{N} - 1 \Rightarrow$$

$$N_P = -N_{th} \ln\left(\frac{N_i}{N}\right) - N + N_i \Rightarrow$$

$$N_P^{peak} = N_{th}\left[\frac{N_i}{N_{th}} - \ln\left(\frac{N_i}{N_{th}}\right) - 1\right]$$

- The peak power

$$P_{out}^{peak} = \left(N_P^{peak} h\upsilon V_p\right)\left(v_g \alpha_m\right)$$

- The peak power increases with the value N_i/N_{th}.
  - ✓ Enlarge the difference between high loss and low loss.
  - ✓ Enhance the pump power
  - ✓ Longer upper-level carrier lifetime

$$\frac{dN}{dt} = -\mathrm{v_g} g N_P$$

$$\frac{dN_P}{dt} = \mathrm{v_g} g N_P - \frac{N_P}{\tau_p}$$

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Pulse energy

- Every photon is generated from the drop of one carrier population, so the total photon density of the laser pulse inside the cavity is

$$N_P^{total} = N_i - N_f$$

- The total photon energy inside the cavity is

$$E = N_P^{total} h\upsilon V_p = \left(N_i - N_f\right) h\upsilon V_p$$
$$= E_i - E_f$$

with

$$E_i = N_i h\upsilon V_p;\; E_f = N_f h\upsilon V_p$$

E_i is the stored energy in the material before the lasing pulse

E_f is the residual energy in the material after the lasing pulse

- The output pulse energy

$$E_{out} = \frac{\alpha_m}{\alpha_T} E = \frac{\alpha_m}{\alpha_T} \eta_E E_i$$

- The energy utilization factor

$$\eta_E = \frac{E}{E_i} = \frac{E_i - E_f}{E_i}$$
$$= 1 - \frac{N_f}{N_i}$$

Pulse energy

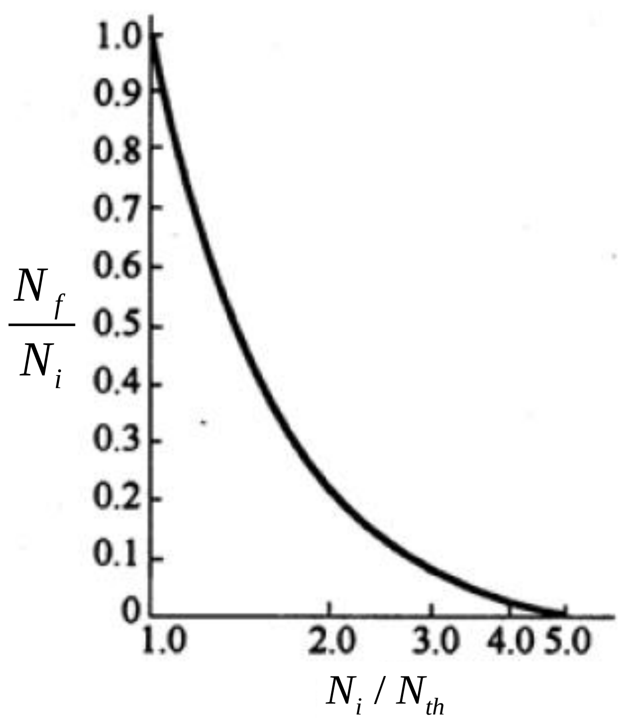
- When the carrier reaches N_f, the photon is zero.

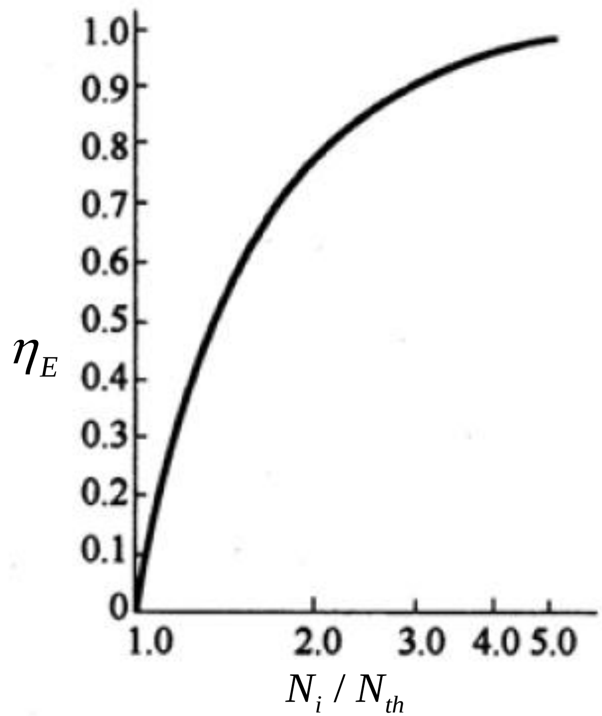
$$N_{Pf} = -N_{th} \ln\left(\frac{N_i}{N_f}\right) - N_f + N_i = 0 \Rightarrow$$

$$\frac{N_f}{N_i} = \frac{N_{th}}{N_i} \ln\left(\frac{N_f}{N_i}\right) + 1$$

$$N_i = R_p \tau_{sp} + \left(N_f - R_p \tau_{sp}\right) \exp\left(-\frac{\tau_{in}}{\tau_{sp}}\right)$$

- The energy utilization factor increases with the ratio N_i/N_{th}.





Pulse duration

❑ The pulse duration is approximately the pulse energy divided by the peak power

$$E_{out} = \frac{\alpha_m}{\alpha_T} E = \frac{\alpha_m}{\alpha_T} \eta_E N_i h\upsilon V_p$$

$$P_{out}^{peak} = N_{th}\left[\frac{N_i}{N_{th}} - \ln\left(\frac{N_i}{N_{th}}\right) - 1\right] h\upsilon V_p \left(v_g \alpha_m\right)$$

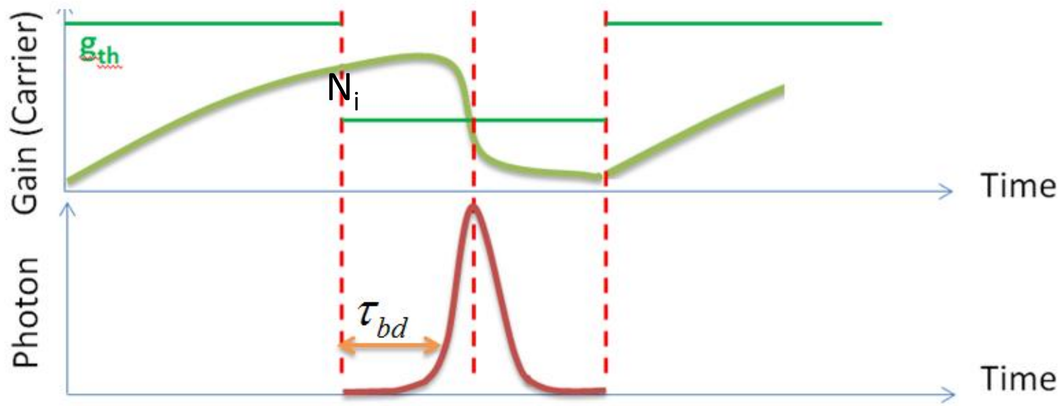
$$\Delta t_d = E_{out} / P_{out}^{peak}$$

$$= \tau_p \eta_E \frac{N_i / N_{th}}{N_i / N_{th} - \ln\left(N_i / N_{th}\right) - 1}$$

❑ The pulse duration depends only on the photon lifetime and the ratio N_i/N_{th}. The laser pulse usually travels several round in the cavity. (See the argument on pp. 332)

Pulse build-up time

- The pulse build-up time is usually defined as the time needed for the laser power reaches 10% of the peak power starting from the Q switching.



- The pulse build-up time is usually defined as the time needed for the laser power reaches 10% of the peak power starting from the Q switching.

$$\frac{dN_P}{dt} = v_g g N_P - \frac{N_P}{\tau_p} \Rightarrow$$

$$N_P(t) = N_{Pi}\exp\left[\left(v_g g - \frac{1}{\tau_p}\right)t\right]$$

$$v_g g - \frac{1}{\tau_p} \approx v_g \sigma N_i - \frac{1}{\tau_p}$$

$$= v_g \sigma N_{th}\frac{N_i}{N_{th}} - \frac{1}{\tau_p}$$

$$= \frac{1}{\tau_p}\left(\frac{N_i}{N_{th}} - 1\right)$$

$$N_P(t) = N_{Pi}\exp\left[\left(\frac{N_i}{N_{th}} - 1\right)\frac{t}{\tau_p}\right]$$

$$\frac{1}{10}N_P^{peak} = 1 \times \exp\left[\left(\frac{N_i}{N_{th}} - 1\right)\frac{t}{\tau_p}\right] \Rightarrow$$

$$\tau_{bd} = \frac{\tau_p}{N_i / N_{th} - 1}\ln\left(\frac{1}{10}N_P^{peak}\right)$$

Examples 8.4

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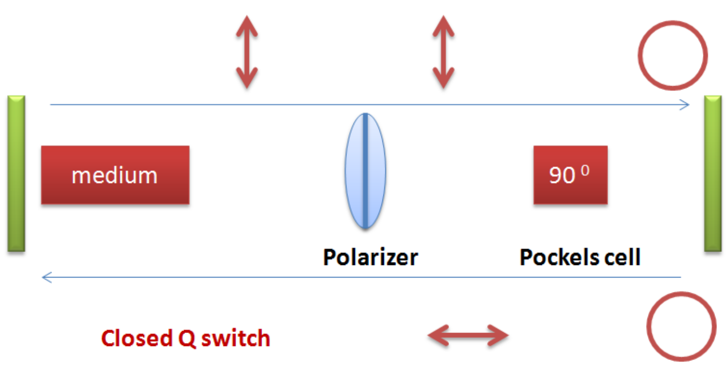
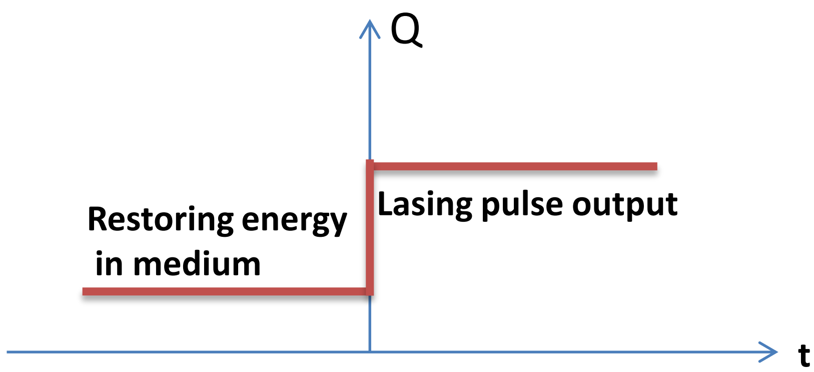
Cavity dumping technique

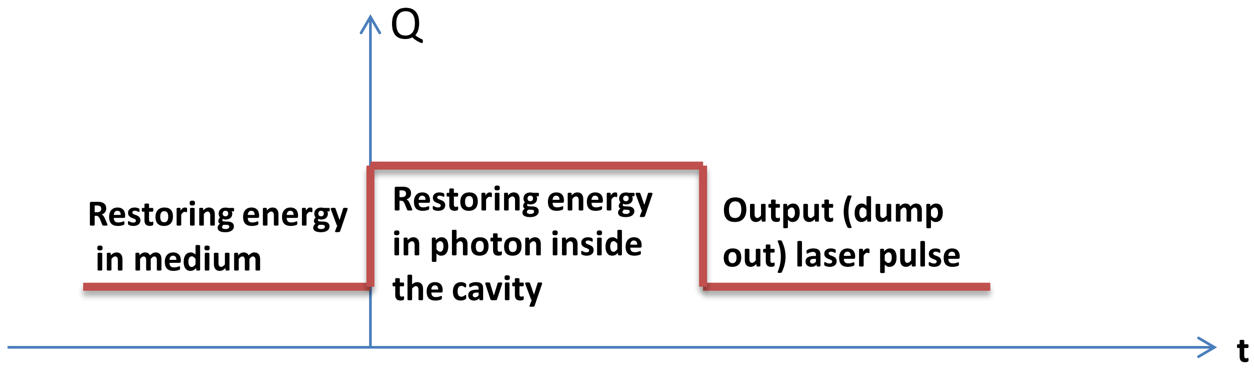
❑ In the Q switching technique, the medium restores energy when the cavity Q is low. Once the cavity Q is switched to a high level, the laser pulse will build up in the cavity, meanwhile the pulse is output from the cavity. The laser pulse will oscillates several round trips inside the cavity before the end, so the pulse duration is usually tens of nanosecond.



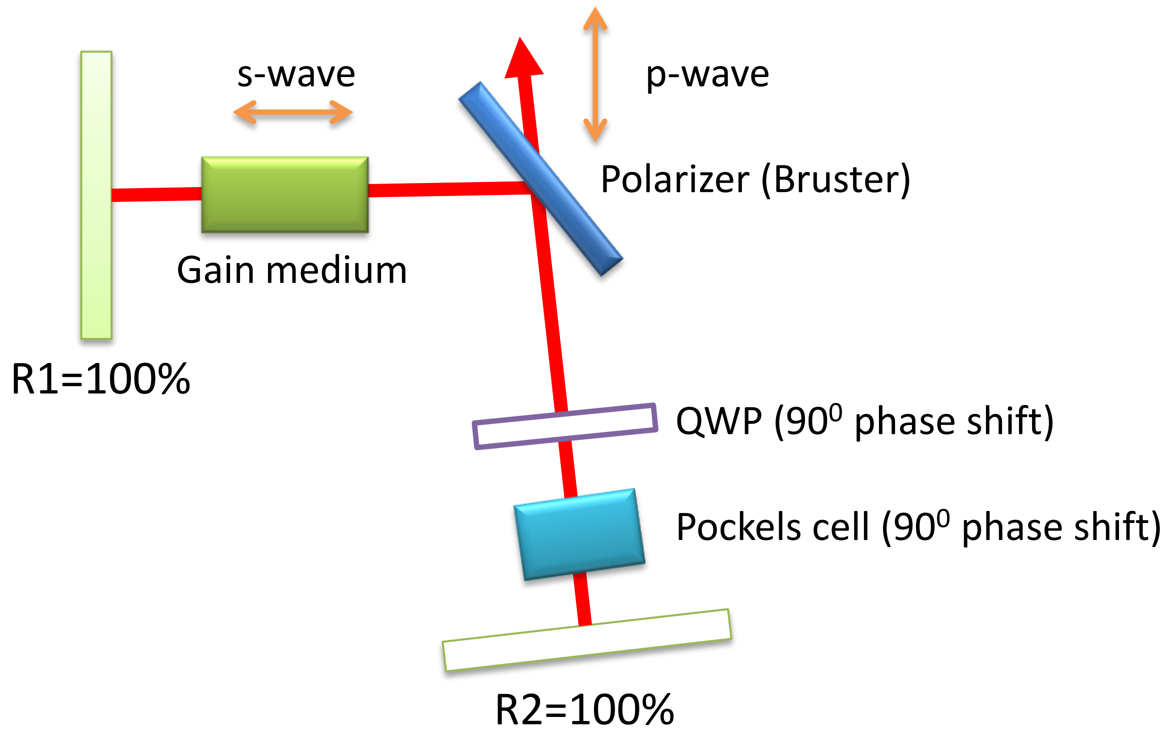


Cavity dumping technique

❑ In the cavity dumping technique, the medium restores energy when the cavity has a low Q, once the Q value is switched to a high level, the laser oscillates in the cavity but no output (0%). When the Q value is switched back to the low level, the photons are dumped out (T=100%) of the cavity within only one round trip, so the pulse duration is only several nanosecond.



Cavity dumping technique

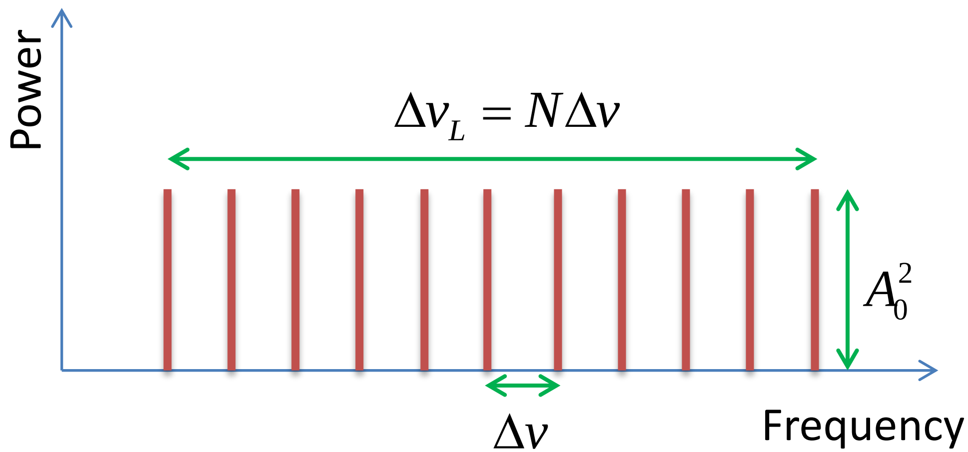


- ❑ When the Pockels cell is off (no voltage). The lasing polarization is s-wave, totally reflected by the Bruster polarizer to the quarter-wave plate (QWP), a round pass through the QWP induces 180^0 phase shift, so the polarization becomes p-wave and the output from the polarizer is 100%. The cavity has a low Q to restore energy in the medium.
- ❑ When the Pockels cell is on. The s-wave double pass the QWP and the Pockesl cell, which induces 360^0 phase shift, thus the polarization is still s-wave and the output from the polarizer is 0%. The cavity has a high Q to restore energy in the photon.
- ❑ When the Pockels cell is off. The output is 100%, the laser pulse is dumped out in one round trip.

- Mode locking technique

Mutimode laser

- Consider a laser with N longitudinal modes, the mode amplitude is A_0, the mode spacing is Δv, the mode (oscillating) bandwidth is $\Delta v_L = N\Delta v$, determined by the gain bandwidth. The phase of each mode varies randomly.



- Consider 3 modes

$$\boxed{\begin{aligned} E_0(t) &= A_0 \exp\left[j\omega_0 t + j\varphi_0(t) \right] \\ E_1(t) &= A_0 \exp\left[j(\omega_0 + \Delta\omega)t + j\varphi_1(t) \right] \\ E_{-1}(t) &= A_0 \exp\left[j(\omega_0 - \Delta\omega)t + j\varphi_{-1}(t) \right] \end{aligned}}$$

- The total electric field is

$$\boxed{E(t) = E_0(t) + E_1(t) + E_{-1}(t)}$$

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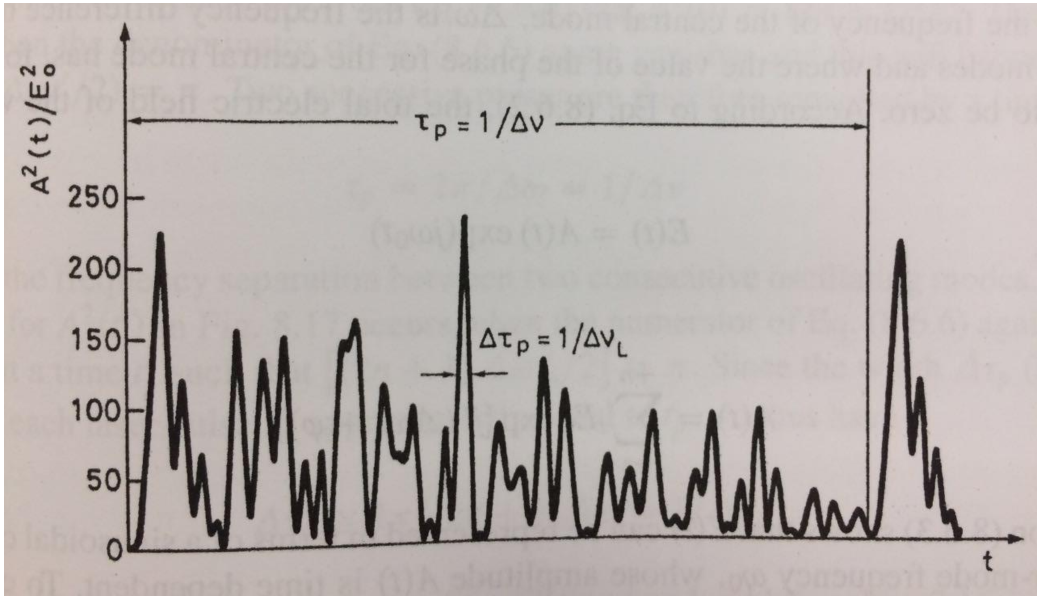
Multimode laser

- The total power is (proportional to)

$$\left|E(t)\right|^2 = E(t)E^*(t)$$
$$= 3A_0^2 + 2A_0^2\left[\cos(\Delta\omega t + \varphi_1 - \varphi_0) + \cos(\Delta\omega t + \varphi_0 - \varphi_{-1}) + \cos(2\Delta\omega t + \varphi_1 - \varphi_{-1})\right]$$

- Since the phases vary randomly, the total light power shows a random time behavior.



✓ The waveform is periodic with a period

$$\tau_p = 1/\Delta v$$

✓ Each light pulse duration of the random waveform roughly equals to

$$\Delta\tau_p = 1/\Delta v_L$$

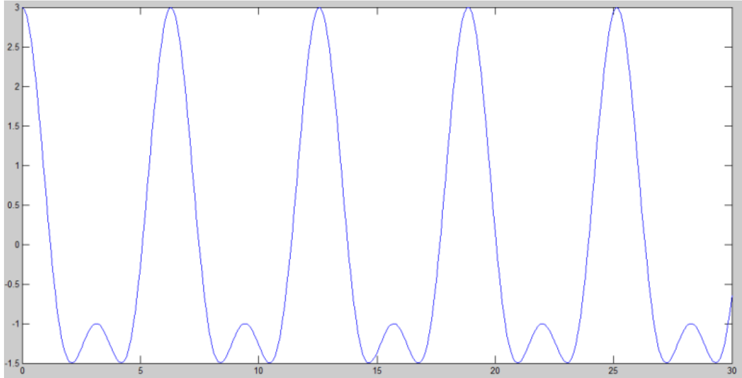
✓ The pulse duration is on the order of picoseconds or less, which can not be detected by the PD. Thus, the monitored value is an time averaged value.

✓ The average value is the sum of each mode power, proportional to N*A_0^2.

Mode-locked laser

- If the phase of each mode has a definite phase relation (eg. difference), rather than varies randomly. The multimode laser is then mode-locked laser. The phenomenon is called mode locking, what locked is the phase.

$$\varphi_l - \varphi_{l-1} = \varphi; \; l = -n, \; -n+1 \ldots n-1, \; n$$



Time series of three locked modes

- The central mode frequency is w_0, and the phase is set to be 0. The total electric field is

$$\begin{aligned} E(t) &= \sum_{l=-n}^{+n} A_0 \exp\left[j(\omega_0 + l\Delta\omega)t + jl\varphi \right] \\ &= \left(\sum_{l=-n}^{+n} A_0 \exp\left[jl(\Delta\omega t + \varphi) \right] \right) \exp\left(j\omega_0 t \right) \\ &= A(t) \exp\left(j\omega_0 t \right) \end{aligned}$$

- Therefore, E(t) can be represented by a sinusoidal carrier wave at the center mode frequency w_0, whose amplitude is time dependent.

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Mode-locked laser

- The time dependent amplitude

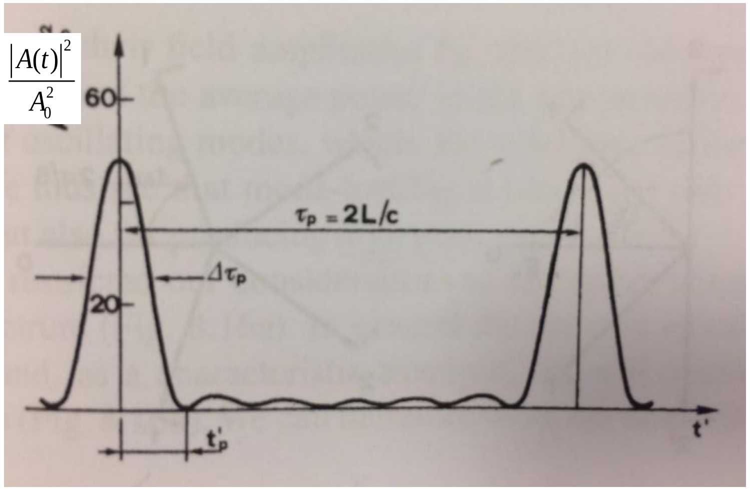
$$A(t)=\sum_{l=-n}^{+n} A_0 \exp\left[\, jl(\Delta\omega t+\varphi)\right]$$

$$A(t')=\sum_{l=-n}^{+n} A_0 \exp\left(jl\Delta\omega t'\right)$$

$$=A_0\,\frac{\sin\left[(2n+1)\dfrac{\Delta\omega t'}{2}\right]}{\sin\left[\dfrac{\Delta\omega t'}{2}\right]}$$

- Introduce a new time reference

$$\Delta\omega t'=\Delta\omega t+\varphi$$



- The laser produces a train of evenly spaced light pulses due to the interaction of phase locked modes. The pulse maxima occur at those times for which the denominator becomes zero.

$$\frac{\Delta\omega t'}{2}=m\pi$$

- Therefore, the successive pulse time interval is

$$\tau_p=\frac{2\pi}{\Delta\omega}=\frac{1}{\Delta\nu}=\frac{2n_r L}{c}$$

Mode-locked laser

- The first maximum occurs at t'=0, using the approximation sin x~x

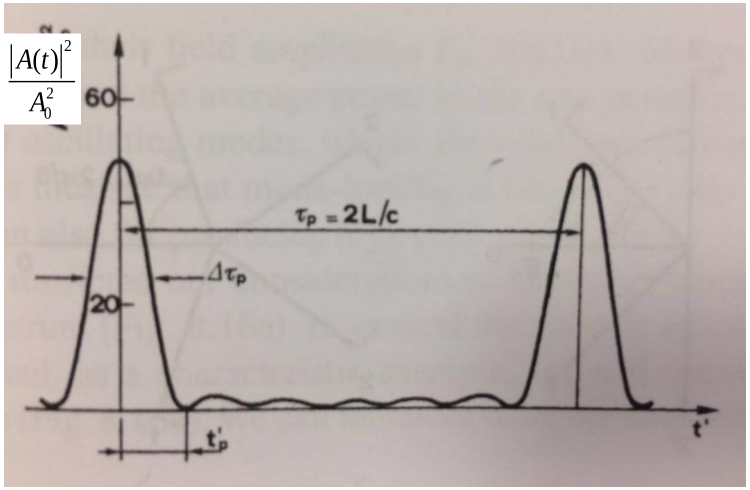
$$\boxed{A(0) = (2n+1)A_0}$$

- The peak power is proportional to

$$\boxed{A^2(0) = (2n+1)^2 A_0{}^2}$$

- The first zero occurs at a time t_p' such that

$$\boxed{(2n+1)\frac{\Delta\omega t_p'}{2} = \pi}$$



- The pulse width (FWHM) is approximately equal to t_p'

$$\boxed{\Delta\tau_p \approx t_p' = \frac{2\pi}{(2n+1)\Delta\omega} = \frac{1}{\Delta\nu_L}}$$

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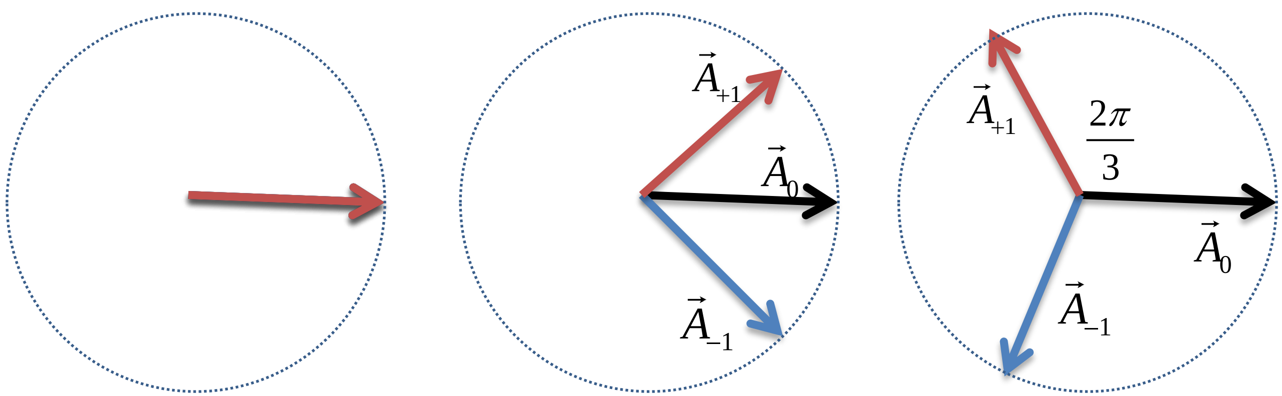
Mode-locked laser

- The field components can be represented by vectors in the complex plane, the l-th component corresponds to a complex vector of amplitude A_0, which rotates at the angular velocity $l\Delta\omega$, refer to argument on pp. 342.

$$\vec{A}(t') = \vec{A}_0(t') + \vec{A}_{+1}(t') + \vec{A}_{-1}(t')$$

$A(t') = Max$ $\qquad$ $A(t') = 0$



$t' = 0$ $\qquad$ $t' > 0$ $\qquad$ $t' = t_p'$

$\Delta\omega t' = 2m\pi$ $\qquad$ $\Delta\omega t_p' = \frac{2\pi}{2n+1}$

Mode-locked laser

❑ In mode locked laser, the pulse duration is equal to the oscillating bandwidth (mode number), which is determined by the gain width. The pulse duration ranges from a few ps down to a few fs.

✓ In solid-state and semiconductor lasers, the pulse duration is a few picosecond.

✓ In dye or tunable solid-state lasers, the gain linewdith is at least 100 times larger, which leads to a pulse duration of a few femtosecond, such as ~25 fs dye laser, ~7 fs Ti:sapphire laser.

✓ In gas lasers, the gain linewidth is narrower on the order of a few GHz, leading to a long pulses down to ~100 ps.

❑ In the mode locked laser, the peak power of each pulse is proportional to $(2n+1)^2A_0^2$, while a laser with modes of random phase has an average power proportional to $(2n+1)\,A_0^2$. Therefore, mode-locking laser produces high peak power.

❑ Therefore, mode-locked laser with phase-locked modes provides pulse trains of both short duration and high peak power.

❑ Requirements: mutimode, equal mode spacing, fixed mode phase relation

Mode-locked laser

- The spectral envelope of lasers are usually of Gaussian distribution due to the inhomo. gain broadening. The amplitude A_l of the *l-th* mode is described as:

$$A_l^2 = A_0^2 \exp\left[-\ln 2 \left(\frac{2l\Delta\omega}{\Delta\omega_L} \right)^2 \right]$$

$\Delta\omega_L$ represents the FWHM bandwidth of the spectrum.

- The time-dependent amplitude of the total electric field is

$$A(t') = \sum_{l=-\infty}^{+\infty} A_l \exp(jl\Delta\omega t')$$

- That is, the electric field amplitude *A(t)* in the time domain is the Fourier transform of the spectral amplitude A_l in the optical domain.

- Each pulse intensity is a Gaussian function of time

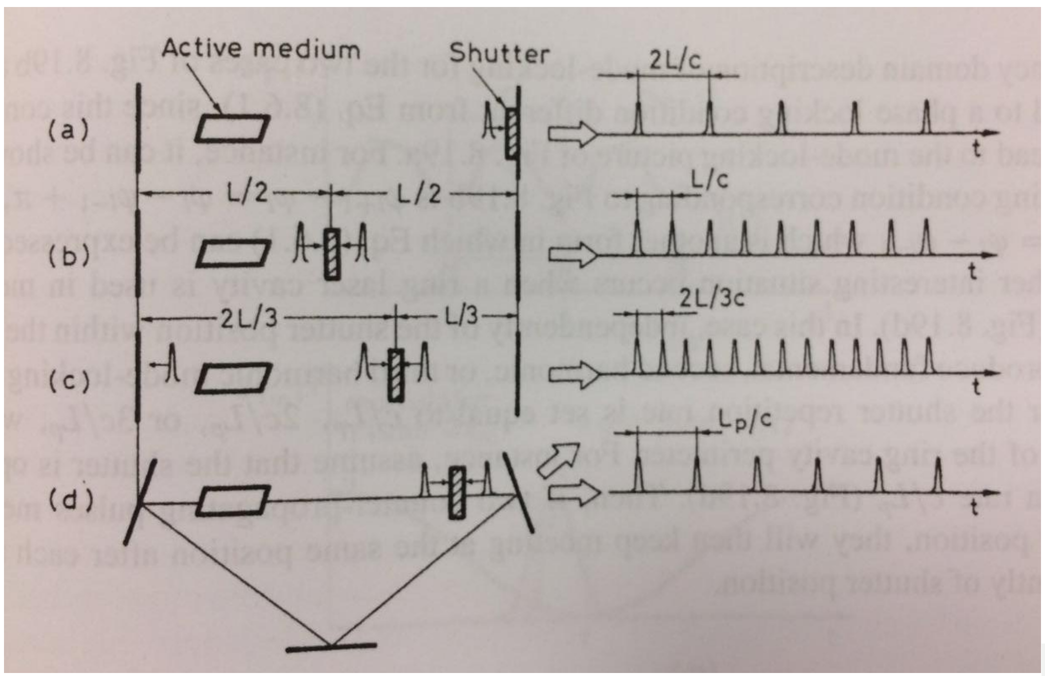
$$A^2(t) \propto \exp\left[-\ln 2 \left(\frac{2t}{\Delta\tau_p} \right)^2 \right]$$

- The pulse duration is

$$\Delta\tau_p = \frac{4\ln 2}{\Delta\omega_L} \approx \frac{0.441}{\Delta\nu_L}$$

Mode-locked laser (time domain)

❑ In mode-locked laser, the time interval τ_p of two consecutive pulses is equal to the cavity round-trip time T_{rt}. This can be achieved by inserting a fast shutter at one end of the cavity. The shutter is periodically opened with a period T_{rt}, and the opening duration is $\Delta\tau_p$. This is the case shown in (a). The laser oscillation behavoir can be visualized as a single ultrashort pulse propagates back and forth within the cavity. When the pulse goes to the mirror, there is one pulse output. Therefore, $\tau_p = T_{rt.}$ This is refered as **fundamental mode-locking**. Note that the shutter is able to lock the phase of each mode.



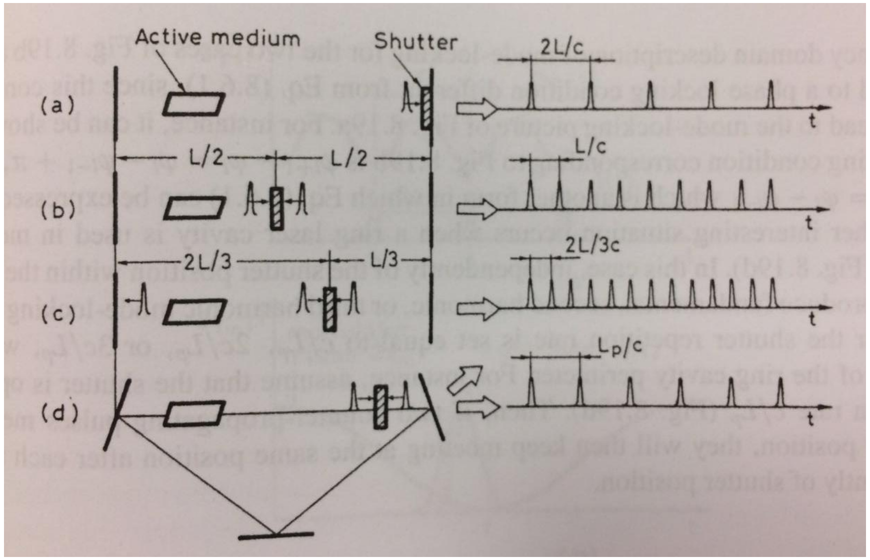
Mode-locked laser (time domain)

- If the shutter is placed at the middle of the cavity, and has an open period of $T_{rt}/2$, two ultrashort pulses present in the cavity such that the two pulses cross each other when the shutter is open. Then, mode locking is achieved with pulse interval time $T_{rt}/2$. This is called the second harmonic mode locking. (二次谐波锁模)
- If the shuter is place at one third length of the cavity, and has an open period of $T_{rt}/3$, three pulses present in the cavity such that two of them cross each other when the shutter is open. Then, mode locking is achieven with pulse intercal time $T_{rt}/3$. This is called the third harmonic mode locking. (三次谐波锁模)

- In the ring cavity, the pulse repetition rate only depends on the shutter open period, independent of the shutter position within the cavity.

- The phase locking condition:

$$\text{Fundamental } \varphi_{l+1}-\varphi_l=\varphi_l-\varphi_{l-1}+2\pi$$
$$\text{Second } \varphi_{l+1}-\varphi_l=\varphi_l-\varphi_{l-1}+2\pi/2$$
$$\text{Third } \varphi_{l+1}-\varphi_l=\varphi_l-\varphi_{l-1}+2\pi/3$$



❑ Mode locking methods

- ✓ Active mode locking
- ✓ Passive mode locking

Mode-locking methods

❑ Like Q-switching techniques, active mode locking requires elements driven by current/voltage sources. Passive mode locking employs nonlinear optical effects such as saturation of saturable absorber or nonlinear refractive index change of a certain material.

❑ Active mode locking has three main types:

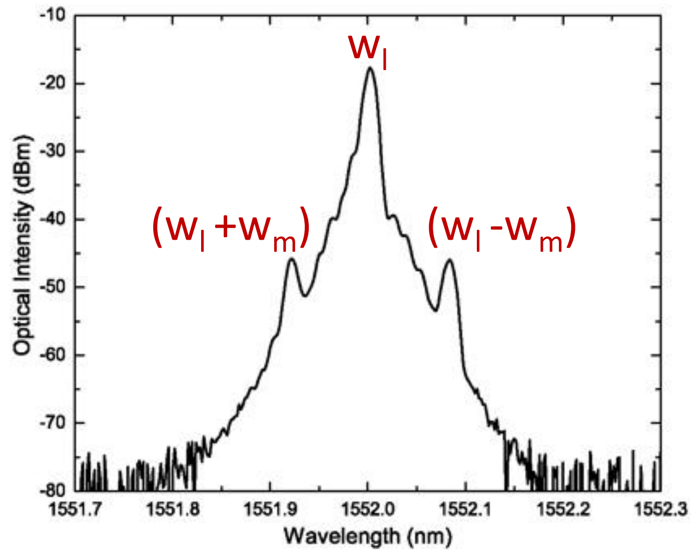
- ✓ Amplitude mode (AM) locking using an amplitude modulator
- ✓ Frequency mode (FM) locking using a phase modulator
- ✓ Gain modulation by synchronous pumping (seldom)

AM mode locking

- If the multimode laser's amplitude is modulated with a small amplitude *m* at a frequency w_m,

$$
\begin{aligned}
E_l(t) &= \left[A_0 \exp(j\omega_l t + j\varphi_l)\right]\left[1 + m\cos(\omega_m t)\right] \\
&= \left[A_0 \exp(j\omega_l t + j\varphi_l)\right]\left[1 + \frac{m}{2}\exp(j\omega_m t) + \frac{m}{2}\exp(-j\omega_m t)\right] \\
&= A_0 \exp(j\varphi_l)\left[\exp(j\omega_l t) + \frac{m}{2}\exp\left[j(\omega_l + \omega_m)t\right] + \frac{m}{2}\exp\left[j(\omega_l - \omega_m)t\right]\right]
\end{aligned}
$$

- Therefore, the modulated signal has three components: the carrier wave at w_l, and two small sidebands (pure sine waves) with frequencies slightly above and below the carrier frequency $(w_l + w_m)$ and $(w_l - w_m)$.



AM mode locking

- If the modulation frequency equals to the mode spacing $\omega_m = \Delta\omega$, the modulation sidebands will coincides with the adjacent mode frequencies of the laser resonator.
- Thus, the equations for cavity modes become coupled, i. e., the field equation of a given cavity mode will contain two contributions arising from the modulation of the adjacent modes. The mode coupling mechanism then **lock the mode phases**.
- The modulator modulates the cavity loss with a period $T = 2\pi / \Delta\omega$, which equals to the cavity round-trip time. The stable steady-state condiction corresponds to light pulses passing through the modulator at the times t_{min} when a minimum loss of the modulator occurs. As such, the pulse will return to the modulator after the time T, when the loss is again at a minimum.

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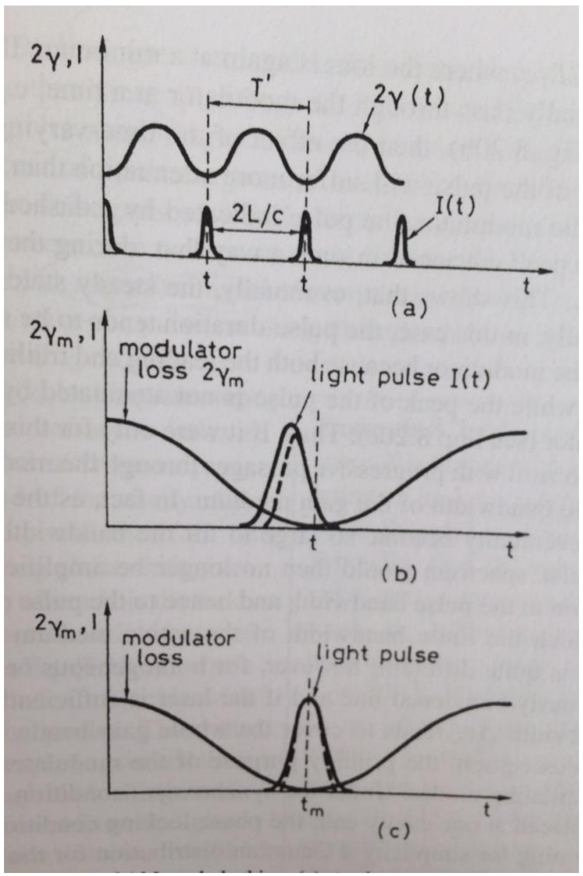
AM mode locking

- If the pulse reaches the modulator at a time slightly deviates from the the time t_{min}, the leading (shorter) or the trailing (longer) edge of the pulse will suffer more attenuation. In the following round trips, the pulse peak will move closer to the minimum loss time t_{min}. Eventually, the steady-state situation will be reached.



- Once the pulse peak is located at the time t_{min}, the pulse duration tends to be shortened each time the pulse passes through the modulator, because both the leading and the trailing edge of the pulse are somewhat attenuated while the peak of the pulse is not attenuated. Therefore, the pulse duration tends to zero with progressive passages through the modulator.

AM mode locking

- On the other hand, a short pulse duration corresponds to a broad sprectum, which is physically limited by the finite gain bandwidth. In the frequency domain, the swings of the pulse spectrum outside the gain bandwidth wont be amplified. This is the fundamental limitation to the pulse spectrum bandwidth, and hence to the pulse duration.

- In the inhomogeneous broadening medium, the mode oscillating bandwidth is rougly the gain bandwidth, if the laser is pumped sufficiently above the threshold. The primary purpose of the modulator is only to lock the phases of the oscillating modes. Assuming the modes envelope is of Gaussian distribution, the AM modulator is located at one end of the cavity, the modulation frequency is $\omega_m = \Delta\omega$, then the pulse duration is

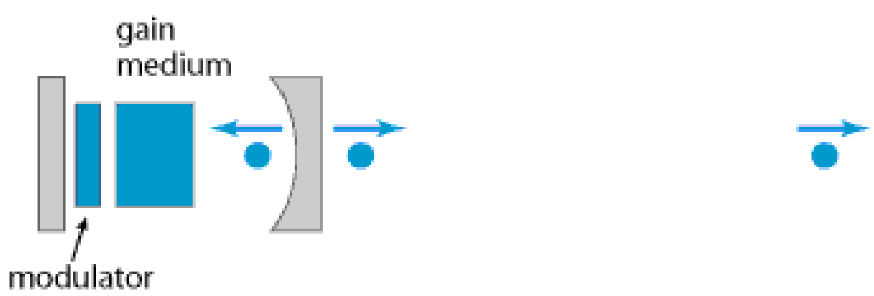
$$\boxed{\Delta\tau_{\mathrm{p}} \approx 0.441 / \Delta v_0^*}$$

AM mode locking

- In the homogeneous broadening medium, the time-varying loss of the AM modulator narrows the pulse duration, and thus broadens the spectrum, while the gain bandwidth limits the pulse duation. The pulse duration is much longer than the inverse of the gain bandwidth. The pulse intensity is also of Gaussian profile with a pulse duration given by

$$\boxed{\Delta\tau_{\mathrm{p}} \approx 0.45/\sqrt{\Delta\nu\Delta\nu_0};\ \Delta\nu << \Delta\nu_0}$$

- Therefore, the mode-locking pulse duration of inhomo broadening medium is much shorter than homo broadening medium for a similar broadening linewidth.
- For a (pulsed) and high gain laser, AM mode locking is achieved by a Pockels cell (EO) amplitude modulator. For a cw and low gain laser, it is achieved by a AO modulator, owing to its lower insertion loss. Here it is standing sound wave in the AO modulator, in comparison with the traveling sound wave in the Q switch.



Examples 8.7

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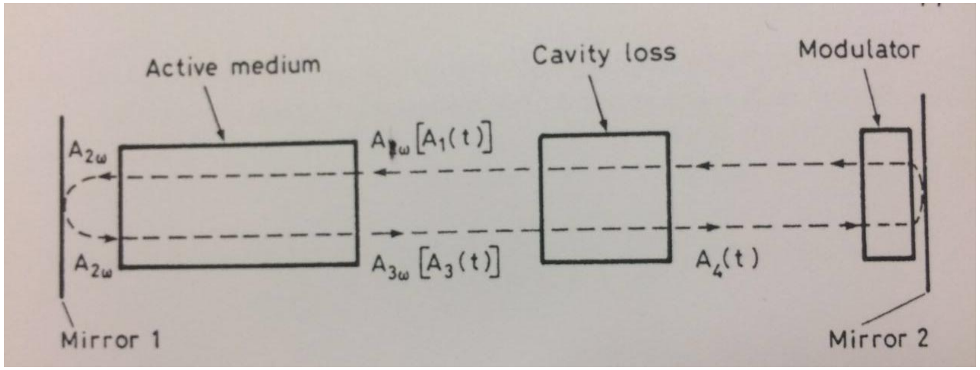
AM mode locking theory

- In the time domain, the mode locking pulse reproduces itself after each round trip.
- Assuming a homogeneous broadening medium with a lifetime much longer than the cavity round trip time. At the gain peak, the saturated gain coefficient g with a light intensity I is

$$\boxed{g = \frac{g_0}{1 + I / I_s}}$$

- The AM modulator is very thin and is placed at mirror 2. Thus, a single pulse travels back and forth within the cavity. At any given position in the cavity, the electric field of the pulse in the time domain is given by



$$\boxed{E(t) = A(t)\,\mathrm{e}^{j(\omega_0 t - \phi)}}$$

The equation is what is the solution of A(t)?

- The spectral amplitude of the pulse is obtained from the Fourier transfrom

$$\boxed{A(\omega - \omega_0) = \int_{-\infty}^{+\infty} A(t)\,\mathrm{e}^{-j(\omega-\omega_0)t}\, dt}$$

$$\boxed{A(t) = \int_{-\infty}^{+\infty} A(\omega - \omega_0)\,\mathrm{e}^{j(\omega-\omega_0)t} d(\omega - \omega_0)}$$

Gain medium contribution

- For one single pass of the gain medium (length *l*) from A_1 to A_2:

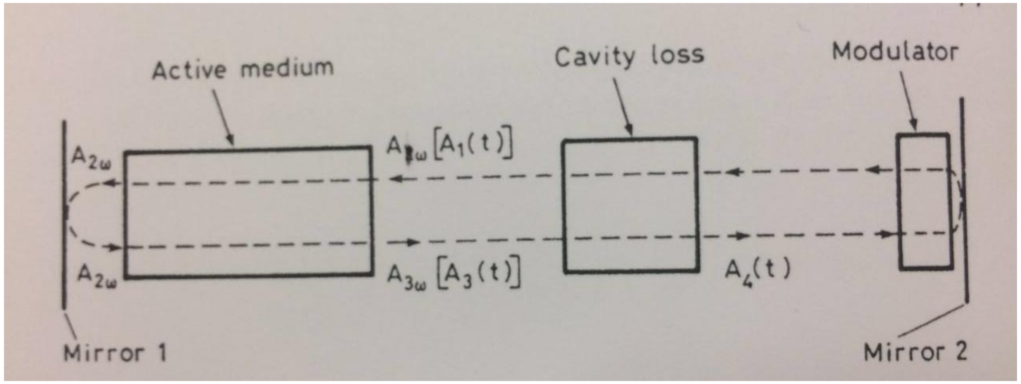
$$A_{2\omega} = A_{1\omega} \exp\left[\frac{gl/2}{1+2j(\omega-\omega_0)/\Delta\omega_0}\right] \exp\left[-j\frac{\omega}{c}n_r l\right]$$

- The power gain:

$$G(\omega) = \left|\frac{A_{2\omega}}{A_{1\omega}}\right|^2 = \exp\left[g(\omega)l\right]$$

with the Lorzentian gain coefficient

$$g(\omega) = \frac{g}{1+\left[2(\omega-\omega_0)/\Delta\omega_0\right]^2}$$



- The spectral width of the pulse is much narrower than the gain bandwith:

$$A_{2\omega} \approx A_{1\omega} \exp\left\{\left[1-\left(\frac{2(\omega-\omega_0)}{\Delta\omega_0}\right)^2\right]\left(\frac{gl}{2}\right)\right\} \exp\left[-j\left(\frac{\omega}{c}n_r l + \frac{\omega-\omega_0}{\Delta\omega_0}gl\right)\right]$$

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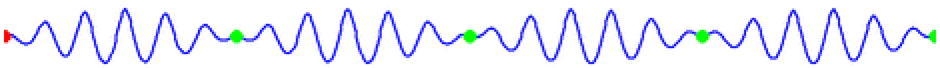
Gain medium contribution

- Then, the phase delay of the pulse after passing the gain medium

$$\phi = \frac{\omega}{c} n_r l + \frac{\omega - \omega_0}{\Delta\omega_0} g l$$

- The time delay of the pulse is determined by the group velocity (amplitude envelope)

$$\tau_d = \frac{l}{v_g} = \frac{d\phi}{d\omega} = \frac{nl}{c} + \frac{gl}{\Delta\omega_0}$$



Group velocity

$$v_g = \frac{d\omega}{dk}$$

Phase velocity

$$v_p = \frac{\omega}{k}$$

- Only consider the absolute value of the complex amplitude, the active medium (single pass) contribution:

$$|A_{2\omega}| \approx |A_{1\omega}| \exp\left\{\left[1 - \left(\frac{2(\omega - \omega_0)}{\Delta\omega_0}\right)^2\right]\left(\frac{gl}{2}\right)\right\}$$

- Therefore, the round-trip pass of the gain medium

$$|A_{3\omega}| \approx |A_{1\omega}| \left\{1 + \left[1 - \left(\frac{2(\omega - \omega_0)}{\Delta\omega_0}\right)^2\right](gl)\right\}$$

Gain medium contribution

- In the time domain, through the inverse Fourier transform:

$$\boxed{A_3(t) \approx \left\{1+(gl)\left[1+\left(\frac{2}{\Delta\omega_0}\right)^2 \frac{d^2}{dt^2}\right]\right\} A_1(t)}$$

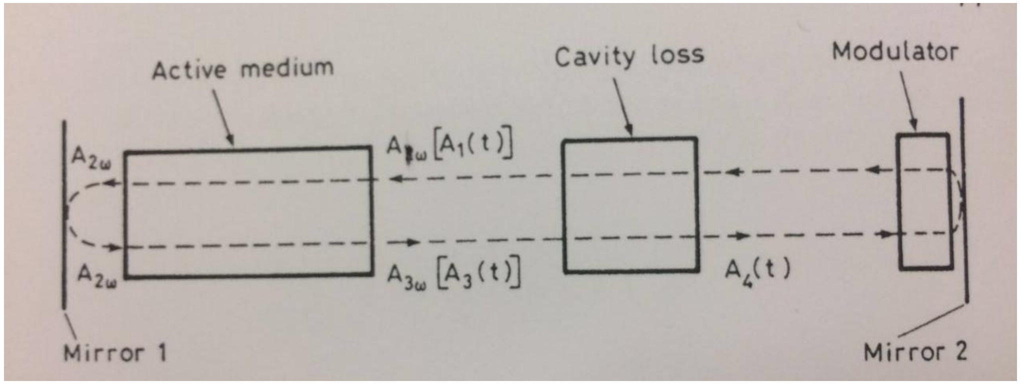
- Then, the round-trip contribution of the gain medium is given by an operator

$$\boxed{\hat{T}_g \approx \left\{1+(gl)\left[1+\left(\frac{2}{\Delta\omega_0}\right)^2 \frac{d^2}{dt^2}\right]\right\}}$$

Cavity loss contribution

- The cavity loss arises from the finite mirror reflectivity and internal losses. The single-pass amplitude is

$$\boxed{A_4(t) = \exp\left(-\frac{\alpha_T L}{2}\right) A_1(t)}$$



- Thus, the round-trip contribution of the cavity loss to the field amplitude is given by an operator

$$\boxed{\hat{T}_\alpha = \exp(-\alpha_T L) \approx 1 - \alpha_T L}$$

AM modulator contribution

- The time varying loss coefficient of the modulator is characterized by

$$\alpha_{md}(t) = \alpha_{md0}\left(1 - \cos(\omega_m t)\right)$$

- Then, the single-pass field amplitude through the modulator is

$$A_5(t) = \exp\left(-\frac{\alpha_{md}(t) l_m}{2}\right) A_4(t)$$

- Thus, the round-trip contribution of the modulator to the field amplitude is given by an operator

$$\hat{T}_{md} = \exp\left[-\alpha_{md0}\left(1 - \cos(\omega_m t)\right) l_m\right] \approx 1 - \alpha_{md0} l_m \left(1 - \cos(\omega_m t)\right)$$

- The pulse passes through the modulator when the modulator loss is zero, e. g. t=0

$$\hat{T}_{md} \approx 1 - \alpha_{md0} l_m \frac{(\omega_m t)^2}{2}$$

AM mode locking theory

- The pulse reproduction after each round trip requires that

$$\boxed{\hat{T}_{md}\hat{T}_{\alpha}\hat{T}_{g}A(t) = A(t)}$$

$$\left[1-\frac{\alpha_{md0}l_m}{2}(\omega_m t)^2\right]\left[1-\alpha_T L\right]\left\{1+(gl)\left[1+\left(\frac{2}{\Delta\omega_0}\right)^2\frac{d^2}{dt^2}\right]\right\}A(t)-A(t)=0\Rightarrow$$

$$\left\{(gl)\left[1+\left(\frac{2}{\Delta\omega_0}\right)^2\frac{d^2}{dt^2}\right]-\alpha_T L-\frac{\alpha_{md0}l_m}{2}(\omega_m t)^2\right\}A(t)\approx 0$$

- The solution is

$$A(t)=\exp\left(-\frac{\omega_p^2 t^2}{2}\right)$$

with

$$\omega_p^2=\sqrt{\frac{\alpha_{md0}l_m}{2gl}}\left(\frac{\omega_m\Delta\omega_0}{2}\right)$$

$$\hat{T}_g\approx\left\{1+(gl)\left[1+\left(\frac{2}{\Delta\omega_0}\right)^2\frac{d^2}{dt^2}\right]\right\}$$

$$\hat{T}_\alpha=\exp(-\alpha_T L)\approx 1-\alpha_T L$$

$$\hat{T}_{md}\approx 1-\frac{\alpha_{md0}l_m}{2}(\omega_m t)^2$$

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AM mode locking theory

- The pulse duration is given by

$$\boxed{\begin{aligned}\Delta\tau_p &= \frac{2\sqrt{\ln 2}}{\omega_p} \\ &= \left(\frac{2\sqrt{2}\ln 2}{\pi^2}\right)^{1/2}\left(\frac{gl}{\alpha_{md0} l_m}\right)^{1/4}\left(\frac{1}{v_m \Delta v_0}\right)^{1/2} \\ &\approx 0.45\left(\frac{1}{v_m \Delta v_0}\right)^{1/2}\end{aligned}}$$

FM (PM) mode locking

- If the multimode laser's phase is modulated with a small amplitude β at a frequency w_m, two sidebands will be formed for each mode as the AM modulation. When the modulation frequency equals to the mode spacing, all the cavity modes are coupled (phase locked) to each other through the sidebands. The laser is then mode-locked.

$$
\begin{aligned}
E_l(t) &= A_0\left[\exp(j\omega_l t + j\varphi_l)\right]\left[\exp\left(j\beta\sin(\omega_m t)\right)\right] \\
&\approx A_0\left[\exp(j\omega_l t + j\varphi_l)\right]\left[1 + j\beta\sin(\omega_m t)\right] \\
&= A_0\left[\exp(j\omega_l t + j\varphi_l)\right]\left[1 + \frac{\beta}{2}\exp(j\omega_m t) - \frac{\beta}{2}\exp(-j\omega_m t)\right] \\
&= A_0\exp(j\varphi_l)\left[\exp(j\omega_l t) + \frac{\beta}{2}\exp\left(j(\omega_l + \omega_m)t\right) - \frac{\beta}{2}\exp\left(j(\omega_l - \omega_m)t\right)\right]
\end{aligned}
$$

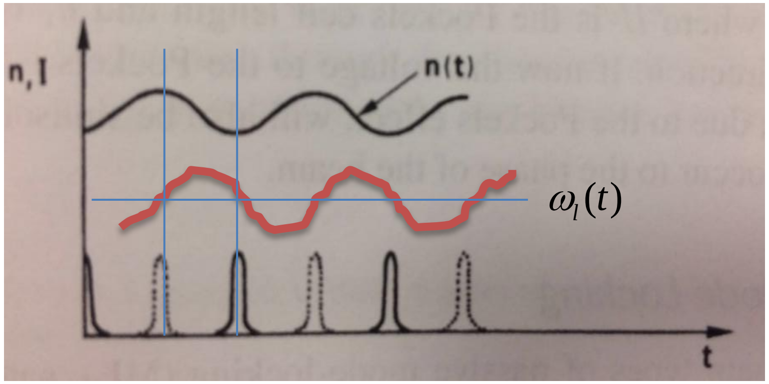
- The above formula used the first-order approximation of the Taylor expansion. In truth, there are an infinite number of sidebands, described by Bessel functions.

$$
E_l(t) = A_0\exp(j\omega_l t + j\varphi_l)\left[J_0(\beta) + \sum_{k=1}^{\infty} J_k(\beta)\exp\left(j\omega_m t\right) + \sum_{k=1}^{\infty}(-1)^k J_k(\beta)\exp\left(-j\omega_m t\right)\right]
$$

FM (PM) mode locking

- If the multimode laser's phase is modulated with a small amplitude β at a frequency w_m, two sidebands will be formed for each mode as the AM modulation. When the modulation frequency equals to the mode spacing, all the cavity modes are coupled (phase locked) to each other through the sidebands. The laser is then mode-locked.

- In the time domain, two stable mode locking states can occur. One is the pulse series occuring at the minimum of the refractive index ($2kn_rL$). The other is at the maximum of the refractive index. (The phase is changed by the refractive index)



- The instantaneous laser frequency is

$$\boxed{\omega_l(t) = \omega_l + \beta\omega_m \cos(\omega_m t)}$$

- Only light pulses passing through the modulator at the refractive index extremes has no frequency shift, while others have somewhat frequency shifts. Since these pulses take the frequency shift each round trip, those will be out of the gain bandwidth and quench. Eventually, only pulses at extremes survive.

FM (PM) mode locking

- FM mode locking usually operates on one of the extreme series. Even a small perturbation can switch one series to another, thus some tehnique is required to stablize the pulse output on only one series.

- For the FM mode locking, a Pockels cell EO phase modulator can be used. Here the light polarization is oriented along one of the birefringence axis, rather than 45^0 angle offset as in a Q switch.

$$\boxed{\Delta\varphi(t) = \frac{2\pi}{\lambda} L \Delta n_r(t)}$$

❑ Mode locking methods

- ✓ Active mode locking
- ✓ Passive mode locking

Passive mode locking

- Passive mode locking methods include

✓ Fast saturable absorber ML: using an absorber with very short upper state lifetime

✓ Kerr lens ML: using the self-focusing effect (Kerr effect, nonlinear optical effect)

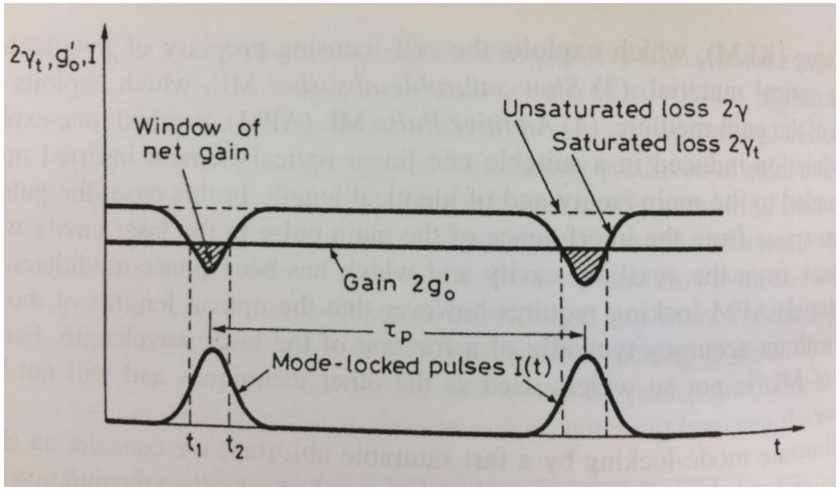
✓ Slow saturable absorber ML: using an absorber with long upper state lifetime together with a fast gain medium.

✓ Additive pulse ML (APM): using the self-phase modulation effect (nonlinear optical effect).

Fast saturable aborber ML

❑ Consider an absorber with a low saturation intensity (large cross section), and with a carrier lifetime much shorter than the duration of the mode locking pulse.

❑ The laser is initially oscillating with unlocked modes. Then, there is a random sequence of light pulses. For the pulse with a low peak intensity, it will suffer a large attenuation when it passes the absorber due to its weak saturation. For the pulse with a high peak intensity, it will suffer a reduced attenuation owing to the strong saturation. If certain conditions are met, this pulse can grow faster than others, and after many round trips the mode locking pulses are envetually established.



❑ Due to the long lifetime of the gain medium, the gain has little change when the pulse passes. The gain saturation is determined by the average intra-cavity laser power.

❑ The pulse is shortened by the time-varying absorber loss, but broadened by the finite gain bandwidth.

Fast saturable absorber ML

- The steady-state pulse amplitude is described by a hyperbolic secant function

$$\boxed{E(t) \propto \mathrm{sec}\,h(t/\tau_{\mathrm{p}})}$$

$$\mathrm{sec}\,h(x) = \frac{2}{e^{x}+e^{-x}}$$

- The pulse duration is related to the pulse time interval.

$$\boxed{\Delta\tau_p \approx 1.76\tau_{\mathrm{p}}}$$

- The pulse duration is calculated by

$$\boxed{\Delta\tau_p \approx \frac{0.79}{\Delta v_0}\sqrt{\frac{g_s}{\alpha_s (I_p / I_s)}}}$$

$$I_s = \frac{hv}{2\sigma\tau_{sp}}$$

where g_s and α_s are saturated value determined by the average intracavity light intensity.

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Fast saturable absorber ML

- The requirements of absorber are firstly a short carrier lifetime (a few ps or shorter to obain a short pulse duration), and secondly a large absorption cross section (10^{-16} cm^2 or larger to botain a low saturation intensity). The most popular saturable absorbers are dye molecules and semiconductors.

- The dye absorbers' carrier lifetime is typically tens of ps. Thus, the absorber remains saturated for a time roughly equal to this lifetime, and ML pulses shorter than a few ps can not be obtained.

- The semiconductor absorber has mutiple mechanisms determining the carrier decay: a) Interband spontaneous emission and nonradiative decay of sub-nanosecond; b) Intraband carrier-carrier scattering of ~0.1 ps; c) Intraband carrier-phonon scattering of ~1.0 ps. The slow mechanism a) leads to a low saturation intensity, while the fast mechanisms b) and c) lead to a short pulse.

Examples 8.8

Saturable absorber ML theory

- The gain operator

$$\hat{T}_g \approx \left\{1+(gl)\left[1+\left(\frac{2}{\Delta\omega_0}\right)^2\frac{d^2}{dt^2}\right]\right\}$$

- The cavity loss operator (without absorber)

$$\hat{T}_\alpha = 1-\alpha_T L$$

- The absorption coefficient of the absorber

$$\alpha_{sa} = \frac{\alpha_{sa0}}{1+I/I_s} \approx \alpha_{sa0}\left(1-I/I_s\right)$$

- The absorber operator

$$\begin{aligned}\hat{T}_{sa} &= \exp\left(-\alpha_{sa} l_{sa}\right)\\ &\approx 1-\alpha_{sa} l_{sa}\\ &\approx 1-\alpha_{sa0} l_{sa}\left(1-I/I_s\right)\end{aligned}$$

Saturable absorber ML theory

- The self reproduction requires

$$\boxed{\hat{T}_{sa}\hat{T}_{\alpha}\hat{T}_{g}A(t)=A(t)}$$

$$\left[1-\alpha_{sa0}l_{sa}\left(1-I/I_s\right)\right]\left[1-\alpha_T L\right]\left\{1+(gl)\left[1+\left(\frac{2}{\Delta\omega_0}\right)^2\frac{d^2}{dt^2}\right]\right\}A(t)-A(t)=0\Rightarrow$$

$$\left\{(gl)\left[1+\left(\frac{2}{\Delta\omega_0}\right)^2\frac{d^2}{dt^2}\right]-\alpha_T L-\alpha_{sa0}l_{sa}\left(1-\frac{I}{I_s}\right)\right\}A(t)\approx 0$$

- The solution is

$$\boxed{A(t)=\frac{A_0}{\cosh(t/\tau_p)}}$$

- The pulse repetition time

$$\boxed{\tau_p=\left(\frac{2g}{\alpha_{sa0}}\right)^{1/2}\left(\frac{2}{\Delta\omega_0}\right)\left(\frac{I_s}{A_0^{\,2}}\right)^{1/2}}$$

- The pulse duration

$$\boxed{\Delta\tau_p=1.76\tau_p}$$

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Mode-locked laser characteristics

TABLE 8.1. Most common media providing picosecond and femtosecond laser pulses together with the corresponding values of: (a) gain linewidth, $\Delta\nu_0$; (b) peak stimulated emission cross-section, σ; (c) upper state lifetime, τ; (d) shortest pulse duration so far reported, $\Delta\tau_p$; (e) shortest pulse duration, $\Delta\tau_{mp}$, achievable from the same laser

Laser medium	$\Delta\nu_0$	$\sigma[10^{-20}\ \mathrm{cm}^2]$	$\tau[\mu s]$	$\Delta\tau_p$	$\Delta\tau_{mp}$
Nd:YAG $\lambda = 1.064\ \mu m$	135 GHz	28	230	5 ps	3.3 ps
Nd:YLF $\lambda = 1.047\ \mu m$	390 GHz	19	450	2 ps	1.1 ps
$Nd:YVO_4$ $\lambda = 1.064\ \mu m$	338 GHz	76	98	<10 ps	1.3 ps
Nd:glass $\lambda = 1.054\ \mu m$	8 THz	4.1	350	60 fs	55 fs
Rhodamine 6G $\lambda = 570\ nm$	45 THz	2×10^4	5×10^{-3}	27 fs	10 fs
Cr:LISAF $\lambda = 850\ nm$	57 THz	4.8	67	18 fs	8 fs
Ti:sapphire	100 THz	38	3.9	6–8 fs	4.4 fs

Homework

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About the scores

- Homework: 10 scores

  Including six chapters (6 scores), and three simulations (Chapter 2 =1 score, Chapter 7=1.5 score, and Chapter 8=1.5 score)

- Project: 10 scores

  Including 3-page report, and oral presentation (English).

- Experiment: 10 scores

- Final exame: 70 scores